Power & Energy Measurement
A Brief Survey

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Contents

Background

This document attempts to briefly explain the issues of measuring and analyzing electrical power and energy (P&E). It contains definitions of the most important quantities and basic relationships among them, including both sinusoidal and nonsinusoidal loads. It introduces the most often used methods of measuring and analyzing P&E. Digital methods are described in greater details.
Treatments of these issues were motivated by increasing pressure of the American government on Energy savings. If we want to save energy we must first know how much and in what manner we consume it.

Executive Order 13123

On June 3, 1999, President Clinton signed Executive Order 13123 “Greening the government through efficient energy management” promoting Government-wide energy efficiency and renewable energy, thereby revoking Executive Orders 12902 and 12759, both of which dealt with reducing the Government's energy use. The new Executive Order strengthens the Government's efforts to pursue energy and cost savings, and raises the energy savings goal to a 30 percent reduction in energy consumption per square foot in nonexempt Federal buildings by the year 2005 and 35 percent by 2010 compared to a 1985 baseline. Industrial and laboratory facilities shall, through life cycle cost effective energy measures, reduce energy consumption per square foot, per unit of production, or per other unit as applicable by 20 percent by 2005 and 25 percent by 2010 relative to 1990.

As the largest energy consumer in the world, the U.S. government's cost- and energy-saving opportunity is enormous. In Fiscal Year (FY) 1996, the government spent nearly $8 billion for its 500,000 buildings, its vehicles, and process energy.
The Federal government of the U.S. is the nation’s single largest purchaser of appliances and equipment, buying over $6 billion worth of energy-using appliances and equipment annually. The Federal Government, as the Nation's largest energy consumer, shall significantly improve its energy management in order to save taxpayer dollars and reduce emissions that contribute to air pollution and global climate change. The Federal Government can lead the Nation in energy efficient building design, construction, and operation and can promote energy efficiency, water conservation, and the use of renewable energy products, and help foster markets for emerging technologies.
The Alliance to Save Energy[W3] estimates that cost-effective energy-efficiency measures would save federal taxpayers $1 billion a year and would require an investment of more than $4 billion.

The Executive Order directs the Secretary of Energy to appoint an advisory committee consisting of representatives from Federal agencies, State governments, energy service companies, utility companies, equipment manufacturers, construction and architectural companies, environmental, energy and consumer groups, and other energy-related organizations. The committee will provide input on Federal energy management, including how to improve use of Energy Savings Performance Contracts and utility energy efficiency service contracts, improve procurement of Energy Star and other energy efficient products, improve building design, reduce process energy use, and enhance applications of efficient energy technologies at Federal facilities. NEMA has sent a letter of nomination to the White House proposing that Timothy Feldman represent electrical manufacturers on this important committee.
The Department of Energy and the Department of Energy’s Federal Energy Management Program (FEMP) are responsible for working with the agencies to ensure that they meet the goals of the E.O. DOE is also responsible for helping federal agencies in identifying products in the upper 25 percent of energy efficiency, providing technical assistance to federal agencies, issuing guidelines to clarify how agencies determine the life cycle costs for investments, and administering and managing the Super Energy Service Companies (ESCO’s) and the Energy Savings Performance Contracts (ESPC’s).
The Executive Order provides opportunity for NEMA members in that it requires Federal agencies to purchase, where life cycle cost effective, Energy Star (and other energy efficient products when acquiring

energy using products. For products where Energy Star (labels are not yet available, agencies shall purchase products that are in the upper 25 percent of energy efficiency as designated by FEMP. NEMA has been working with FEMP to help them develop recommendation sheets to identify products in the upper 25 percent of energy efficiency.

NEMA has been working with the Environmental Protection Agency and DOE to develop specifications to designate products as Energy Star. Another opportunity is that the E.O. requires Federal agencies to evaluate the life cost of products when making purchasing decisions. This requires the Federal purchaser not to look at “first cost“ but to examine the potential life cycle cost savings of projects. Federal agencies are also required to help foster markets for emerging technologies. NEMA will be working with FEMP to ensure they are in contact with the research arm of DOE to identify emerging technologies.
The Executive Order is vague in the identification of specific technologies, i.e. lighting controls. This can be seen as both a hindrance and an improvement in the Executive Order. By not specifying certain technologies, Federal agencies are not required to buy certain products (which could have been omitted from the E.O.), but instead are required to evaluate the life cycle costs effectiveness of products and building improvements. This is great opportunity for NEMA members to sell energy efficient products where they make sense.
DOE / FEMP will provide training and training material to educate Federal purchasers on Energy Savings Performance Contracts, utility energy efficiency service contracts, Energy Star and other energy efficiency products, and life cycle cost analysis. NEMA will continue to work with DOE / FEMP to develop these materials.
NEMA will be working with DOE / FEMP in a multitude of arenas to assist in the identification of energy efficient products, the evaluation of cost effective products for building construction and renovation, and to provide guidance on Energy Savings Performance Contracts.

Under the Super Energy Saving Performance contracts, federal buildings can be retrofitted with: energy efficient lighting, more efficient boilers and chillers, building automation and energy management control systems, heating, ventilating and air conditioning systems, building envelope modifications, chilled water piping and hot water and steam distribution systems, electric motors, refrigeration, electrical or cogeneration systems, renewable energy systems, and electrical distribution systems.

The FY 2000 Department of Energy (DOE) budget request estimates that DOE energy efficiency and renewable energy programs will result in annual energy cost savings of $33 billion by 2010. These energy savings are more than all of the gasoline used annually in the following states: Alaska, Arizona, Florida, Hawaii, Maine, Minnesota, Mississippi, New York, Washington and the District of Columbia.

Electric motors and AC drives

The Department of Energy reported in 1991[W1] that sixty-four percent of all electrical energy was consumed by electric motors. A 1982 Dupont Company report indicated that between 5% and 30% of all motors should use variable speed drive (VSD) (based on payback).
What is a VSD? VSDs are electronic devices that match motor speed to that required for the application. A variable speed drive rectifies incoming AC voltage and current into DC, then inverts the DC back into AC at a different voltage and frequency.

Where are the cost savings? The speed of an induction motor is directly proportional to the frequency of the applied voltage. For centrifugal loads such as fans and pumps, motor power consumption is roughly proportional to the cube of the speed. As a simplified example, if we can slow the motor down to 90% speed, we realize an energy savings of 27%.

Since the early 1980’s, many world manufacturers started to offer so called Energy Efficient Motors with special construction using special materials and technologies.

Electric motors represent 75 percent of industrial power consumption - which makes them the biggest power consumer. This is an enormous energy-saving potential which can be taped.

Energy measuring and monitoring

When an organization makes a substantial commitment to change the direction of its operations in order to give high priority to protecting the environment and reducing energy costs, it still must determine what the net effects of all the associated investment and actions have been. This normally will be required by senior management, who needs to be able to justify short-term higher-cost budgets for capital improvements to produce long-term benefits. Some of these benefits may be relatively easy to quantify. For example, energy and water quantities and associated costs are provided monthly to the facility manager for the facilities under their control, and the cost benefits of some energy and water reduction measures can be readily determined from those bills. Many other issues are not so readily quantified, for example: durability, maintenance, drought-tolerant landscaping, or good indoor air quality.

Determining electrical energy consumption is relatively straightforward, and an ordinary electrical meter is adequate for simple daily, weekly, or other longer period electrical energy determination. If consumption versus time is required, either the manual method of taking frequent meter readings, or automated data collection are necessary. For collection of time-based information, split-core current transducers (CTs) and power transducers (PTs) can be installed without disconnecting power. Data loggers can be used to collect data, which can be downloaded by modem as needed.

Time-based information is essential if electrical demand is to be determined, and, in this case, it is essential to have the appropriate software to determine the "peak" value. In reality the peak is normally a time-averaged value over a sliding 15- or 30-minute time frame. Single or multiple spikes are not indicative of the peak as measured by the local utility.

The use of electricity is expanding, and it is getting complicated to maintain balance between generation and consumption. An effective solution to this problem is possible with the help of automatic control of energy consumption, also known as demand side management.
An increasing number of electrical and electronic loads that produce nonsinusoidal waveforms are being connected to the system. They have started to cause ‘power quality’ problems in the systems and affect the accuracy of the electric power measurement. It is thus necessary to develop and apply new techniques for the control of the electric energy to support the optimal performance of the electric supply network. This development requires that proper information be provided about the quality of the electric supply[11].

Fundamentals of Power and Energy Measurement

The measurement of electrical power and energy consumption is of basic importance, especially in power engineering. The measurement in circuits with nonsinusoidal periodic waveforms is very important, because such waveforms can be often found in heavy-current applications due to using of modern electronically controlled switches for power regulation.
Existing definitions for power terms in alternating current systems work well for single-phase and three-phase system where both voltages and currents are sinusoidal with respect to time.
The active power definition and the active power meters need almost no improvements. For the steady-state operation of single or three-phase systems with linear and balanced loads, supplied with sinusoidal and symmetrical voltages, the currently used definitions are well established and undisputed today. However, for three-phase systems with unbalanced loads, there are multiple power definitions[12].

Instantaneous power is the product of instantaneous current and voltage values

$$p(t) = u(t) \cdot i(t) \qquad [1]$$

Its unit is the Watt [W].
Active power and energy have a clear physical meaning: they reflect the net power flow or energy transfer over the integration period.
The electrical work (consumed energy) during a measurement time t_m is defined as

$$\Delta W = \int_0^{t_m} p(t)dt = \int_0^{t_m} u(t) \cdot i(t)dt \qquad [2]$$

Its unit is the Joule [J]; 1J=1Ws

The average (sometimes called true or active) power P is defined as

$$P = \frac{1}{T}\int_0^{T} p(t)dt = \frac{1}{T}\int_0^{T} u(t) \cdot i(t)dt \qquad [3]$$

Sinusoidal waveform consideration

Sinusoidal Single-phase system

In the case of sinusoidal waveform, the voltages and currents are

$$u(t) = U_m \sin(\omega t + \varphi),\ i(t) = I_m \sin(\omega t) \qquad [4]$$

where U_m (I_m) is a magnitude of the sine-shaped voltage (current) and φ is a phase-shift between $u(t)$ and $i(t)$.
The instantaneous power is

$$p(t) = \frac{U_m I_m}{2}\{\cos\varphi[1 - \cos(2\omega t)] - \sin\varphi \sin(2\omega t)\}$$

The active (true) power in this case is according to [3]

$$P = \frac{U_m I_m}{2}\cos\varphi = UI\cos\varphi \qquad [5]$$

where $U = \frac{U_m}{\sqrt{2}}$, $I = \frac{I_m}{\sqrt{2}}$

are the RMS values of voltage and current.

With a pure resistive load, both the current and voltage are in phase, and the phase shift angle φ is 0 degree and the power factor cosφ is unity. As loads vary from pure inductive through resistive to capacitive, the phase angle varies from -90° to 0° to +90° and the power factor varies from zero to unity.

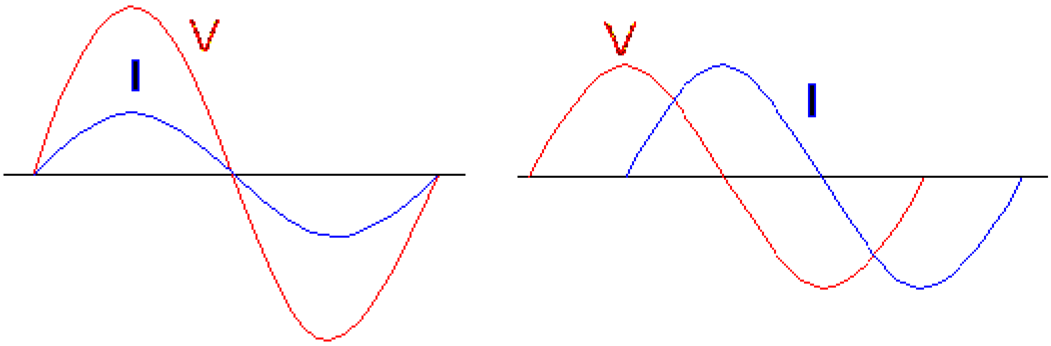


Fig.1: Voltage and current waveforms: in phase (left); voltage leading current (right)

For sinusoidal waveform reactive power Q [VAr] and apparent power S [VA] are also defined

$$Q = UI \sin\varphi \qquad [6]$$

$$S = UI = \sqrt{P^2 + Q^2} \qquad [7]$$

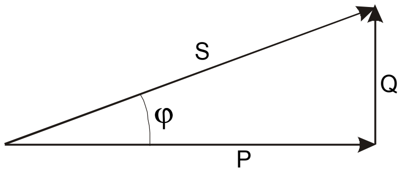


Fig.2: Space diagram of the fundamental power components

The apparent power is sometimes defined as a complex number

$$\overline{S} = \overline{UI}^* = P + jQ\,,\ S = \left|\overline{S}\right| \qquad [8]$$

where I^* is a current phasor conjugate to I phasor.
The AC power drawn from the source is the integral over one cycle of the instantaneous watts values. As shown in Fig.3, during a portion of each cycle power is used by the inductive device (e.g. electric motor), while during other portions of the line cycle, power is actually given back by the inductive device. The portion of the cycle where power is given back by the inductive device is called "negative power". In sinusoidal applications, P = $UI\cos\varphi$ can be seen in Fig.3 above to reflect the true (active) watts during a complete line cycle (360°).

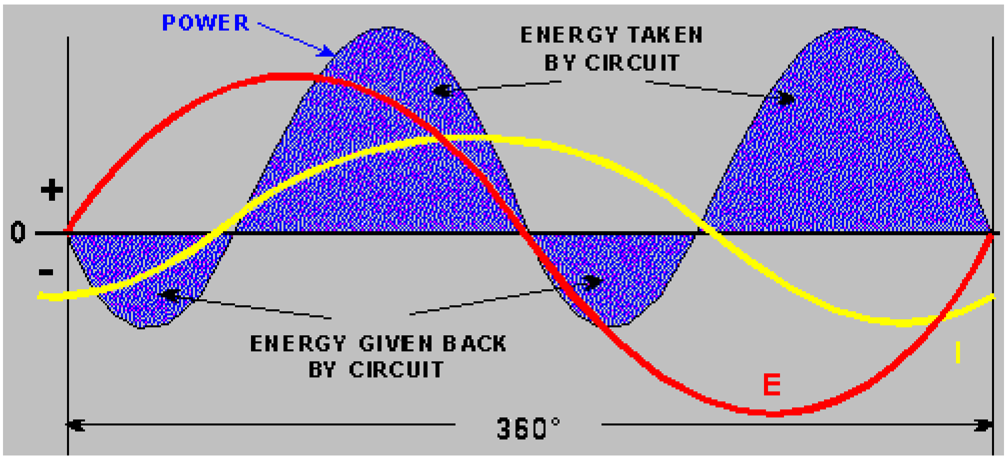


Fig.3: Power cycle with an inductive load

Sinusoidal three-phase system

In the three-phase circuit, the voltage set U_{abc} is usually supposed to be balanced (symmetrical) – Fig.4. The phase voltages U_a, U_b, U_c are of the same magnitude and are phase-shifted 120° with respect to one another. The line voltages U_{ab}, U_{bc}, U_{ca} are defined as

$$U_{ab} = U_a - U_b \,,\; U_{bc} = U_b - U_c \,,\; U_{ca} = U_c - U_a$$

There is

$$|U_a| = |U_b| = |U_c| = U_{LN}$$

$$|U_{ab}| = |U_{bc}| = |U_{ca}| = U_{LL}$$

$$U_{LL} = \sqrt{3} U_{LN} \qquad [9]$$

where index *LL* means Line-to-Line, and *LN* means Line-to-Neutral.

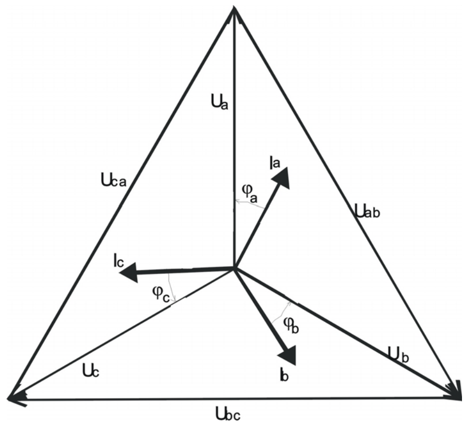


Fig.4: Balanced 3-phase system

3-phase 4-wire systems and 3-phase 3-wire systems are distinguished according to the number of wires used in the system. In the 3-wire system, there is (Kirchhoff's current law)

$$i_a(t) + i_b(t) + i_c(t) = 0 \qquad [10]$$

In 4-wire system (where a neutral wire N or LO is also used):

$$i_a(t) + i_b(t) + i_c(t) + i_N(t) = 0 \qquad [11]$$

If the voltage system is balanced and the load is balanced (if there are the same impedances in all phases), then the current system is also balanced and it is

$$I_a = I_b = I_c = I \,,\; I_N = 0 \,,\; \varphi_a = \varphi_b = \varphi_c = \varphi$$

The total instantaneous power is defined as

$$p(t) = p_a(t) + p_b(t) + p_c(t) = u_a(t) i_a(t) + u_b(t) i_b(t) + u_c(t) i_c(t) \qquad [12]$$

Total active power *P* and reactive power *Q* are defined as

$$P = \frac{1}{T} \int_0^T p(t) dt = 3U_{LL} I \cos\varphi \qquad [13]$$

$$Q = 3U_{LL} I \sin\varphi \qquad [14]$$

The total apparent power of 3-phase balanced sinusoidal system is

$$\overline{S} = 3\overline{U}_{LL} \overline{I}^* = \mathrm{Re}[\overline{S}] + j\,\mathrm{Im}[\overline{S}] = P + jQ \qquad [15]$$

$$|\overline{S}| = S = \sqrt{P^2 + Q^2} \qquad [16]$$

Power factor $\cos\varphi$ of 3-phase and also single-phase system for sinusoidal waveforms can be found as

$$\cos\varphi = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}} = \frac{1}{\sqrt{1 + \tan^2\varphi}} \qquad [17]$$

where φ is a phase shift angle between pure sinusoidal (1-st harmonic) voltage and current per phase.

The average value of $\cos\varphi$ is defined as

$$\cos\varphi_{AV} = \frac{W_P}{\sqrt{W_P^{\ 2} + W_Q^{\ 2}}} \qquad [18]$$

where $W_P = \int_0^T P dt$ is active energy (electrical work), and $W_Q = \int_0^T Q dt$ is reactive energy.

Nonsinusoidal waveform consideration

The present definitions used to evaluate the flow of electric energy in power networks are not adequate for economic studies when nonsinusoidal voltage and current are presented and/or when unbalanced loads cause current and voltage asymmetry in poly-phase systems[13].
Unbalanced, nonsinusoidal load would typically be found in a commercial office building with single-phase, line-to-neutral connected electronic office and computer equipment.

Nonsinusoidal single-phase system

If the waveforms of *u(t)* and *i(t)* are nonsinusoidal, their Fourier series expansion into harmonic components $u_\nu(t)$ and $i_\nu(t)$ is used

$$u(t) = U_0 + \sqrt{2}\sum_{\nu=1}^{\infty} U_\nu \sin(\nu\omega t + \alpha_\nu),\ i(t) = I_0 + \sqrt{2}\sum_{\nu=1}^{\infty} I_\nu \sin(\nu\omega t + \beta_\nu) \qquad [19]$$

where *u(t)* (*i(t)*) is the instantaneous voltage (current), U_0 (I_0) is the average value, U_ν (I_ν) is the RMS value of the ν-th voltage (current) harmonic, and α_ν (β_ν) is the phase angle of the ν-th voltage (current) harmonic.
For RMS values

$$U = \sqrt{\sum_{\nu=0}^{\infty} U_\nu^2},\ I = \sqrt{\sum_{\nu=0}^{\infty} I_\nu^2} \qquad [20]$$

Separating the fundamental component U_1, I_1 from harmonic component U_H, I_H , and neglecting the DC components gives

$$U^2 = U_1^2 + U_H^2,\ I^2 = I_1^2 + I_H^2 \qquad [21]$$

where $U_H^2 = \sum_{\nu=2}^{\infty} U_\nu^2$ and $I_H^2 = \sum_{\nu=2}^{\infty} I_\nu^2$

If the power flow is stationary and if only integer harmonics are involved, the power integral can be written according to [3] as

$$P = P_1 + P_H = U_1 I_1 \cos\varphi_1 + \sum_{\nu=2}^{\infty} U_\nu I_\nu \cos\varphi_\nu \qquad [23]$$

This quantity, the total active power, is measured in many energy meters. The first term of [23], the fundamental active power P_1 is sometimes used too. The difference, the harmonic active power P_H, can be positive or negative and can amount up to a few percent of P_1[14].
The definition of reactive power in nonsinusoidal situations is still heavily debated. Most meters have the following definition implemented:

$$Q_1 = U_1 I_1 \sin\varphi_1 \qquad [24]$$

$$Q_F = \sqrt{S^2 - P^2} \qquad [25]$$

where Q_1 is the so-called fundamental reactive power, while Q_F is known as Fryze reactive power[14].

From [21] we obtain the apparent power S

$$S^2 = (UI)^2 = (U_1 I_1)^2 + (U_1 I_H)^2 + (U_H I_1)^2 + (U_H I_H)^2 \qquad [26]$$

The apparent power S has two components:

$$S^2 = S_1^2 + D^2 = P_1^2 + Q_1^2 + D^2 \qquad [27]$$

where

$$S_1^{\ 2} = (U_1 I_1)^2 = P_1^2 + Q_1^2 \qquad [28]$$

$$P_1 = U_1 I_1 \cos\varphi_1 \qquad [29]$$

$$Q_1 = U_1 I_1 \sin\varphi_1 \,,\ \varphi_1 = \alpha_1 - \beta_1 \qquad [30]$$

Here S_1 is the fundamental apparent power, which is in turn resolved into the fundamental active power P_1 and the fundamental reactive power Q_1.
The nonfundamental apparent power, called the distortion power D, consists of three components

$$D^2 = (U_1 I_H)^2 + (U_H I_1)^2 + (U_H I_H)^2 \qquad [31]$$

The first component is the product of fundamental RMS voltage and harmonic RMS current. Usually this is the dominant term, and may be named Current Distortion Power. The second term, $U_H I_1$, may be called Voltage distortion power, and it is a reflection of the voltage distortion at the observed bus. The third component may be called Harmonic Apparent Power S_H[13].
Generally the value D is good indicator of the level of harmonic "pollution".

We may also define the Nonactive Power N in a conventional way

$$N = \sqrt{S^2 - P^2} \qquad [32]$$

The most expedient figure of merit to quantify the effectiveness of the electric energy flow in a system is the Total Power Factor PF (or λ)

$$PF = \lambda = \frac{P}{S} \qquad [33]$$

The power factor is a number that indicates how much active power P is transmitted out of the maximum possible power S that will cause the same power loss in the supplying equipment and lines[15].

However, isolating P_1, Q_1 and S_1 from the nonfundamental power makes it easy to follow the uncorrupted fundamental power flow of the electric energy, and makes easier the application of engineering economic techniques (such a power factor correction capacitors)[13]. For this reason the Displacement Power Factor DPF remains significant value.

$$DPF = \frac{P_1}{S_1} = \cos\varphi_1 \qquad [34]$$

The DPF is the traditional power factor where the load inductance shifts or displaces the current from the voltage. The true PF (or λ) takes into account the affects of harmonic current. Since harmonic current cannot do useful work, the true PF (or λ) always will be lower than the DPF whenever the line current contains harmonics.

The following terms are also used in literature for describtion of the voltage and current harmonic distortion.

Total harmonic distortion

$$THD_U = \frac{U_H}{U_1} = \frac{\sqrt{U^2 - U_1^2}}{U_1} \,,\ THD_I = \frac{I_H^2}{I_1} = \frac{\sqrt{I^2 - I_1^2}}{I_1} \qquad [35]$$

Distortion factor

$$DF_U = \frac{U_1}{U} = \frac{1}{\sqrt{1 + THD_U^2}} \,,\ DF_I = \frac{I_1}{I} = \frac{1}{\sqrt{1 + THD_I^2}} \qquad [36]$$

The *THD* indicates the percentage of harmonic current with respect to the fundamental (*THDF*) or the total RMS current (*THDR*). A good power meter will show both values so a direct comparison can be made to whatever terminology is used in the ballast specification [12].

Nonsinusoidal three-phase system

For perfectly symmetrical and balanced systems, the notions explained above apply without any restrictions. For unbalanced systems, however, the apparent power definition is still causing a wide range of opinions among the theoreticians as well as practitioners [15].

For unbalanced loads two apparent definitions are prevalent:

The Arithmetic VA:

$$S_A = S_a + S_b + S_c \tag{37}$$

The Vector VA:

$$S_V = \sqrt{P^2 + Q^2 + D^2} \tag{38}$$

where $S_a = U_a I_a = \sqrt{P_a^2 + Q_a^2 + D_a^2}$, $S_b = U_b I_b = \sqrt{P_b^2 + Q_b^2 + D_b^2}$, $S_c = U_c I_c = \sqrt{P_c^2 + Q_c^2 + D_c^2}$

and $P = P_a + P_b + P_c$, $Q = Q_a + Q_b + Q_c$, $D = D_a + D_b + D_c$

Here S_{abc}, P_{abc}, Q_{abc} and D_{abc} represent the per phase apparent, active, reactive and distortion powers, respectivelly.

For balanced loads $S_A = S_V = S = 3UI$ [39]

For unbalanced loads $S_A > S_V$ and the corresponding power factors $PF_A < PF_V$ [12].

The situation is quite complex in the case of 3-phase system. Exepting harmonics the system is negatively affected by unbalanced and nonsymetrical load and time-varying load value.
We have to consider additional kinds of power. So called latent power S_L and ripple power S_R [5].
It is very difficult to measure these powers.
The real apparent power of 3-phase unbalanced nonsymetrical load can be defined as[5]

$$S_\vartheta = \sqrt{P^2 + Q^2 + (3n+1)S_L^2 + S_R^2 + D^2} \tag{40}$$

where n is a neutral/line wire equivalent resistance ratio.
All the last three powers (S_L, S_R, D) are parasite powers, similarly to reactive power Q. Their average value is zero, so they don't contribute to useful energy transmission, but increase distribution system loses.

Definition of an Equivalent Power

Unbalanced systems require additional considerations. In the literature[12, 13, 15], there is increasing support in favor of Equivalent (or System) Apparent Power S_e.

$$S_e = 3U_e I_e \tag{41}$$

where $U_e = \sqrt{\dfrac{U_a^2 + U_b^2 + U_c^2}{3}}$ and $I_e = \sqrt{\dfrac{I_a^2 + I_b^2 + I_c^2}{3}}$ [42]

For a four-wire system, U_{abc} are line-to-neutral RMS voltages. For a three-wire system, the equivalent voltage U_e may be calculated using [42], where U_{abc} are line voltages measured from an artificial neutral point[15] (the star point of the three nonreactive resistors) or

$$U_e = \sqrt{\frac{U_{ab}^2 + U_{bc}^2 + U_{ca}^2}{9}} \tag{43}$$

where the RMS voltages are measured from phase-to-phase.
The expressions [41, 42, 43] suggested by Buchholz, have appeared for the first time in the engineering literature in 1922.

When the load is linear and balanced, then $U_e = U$, $I_a = I_b = I_c = I$, the system apparent power [41] becomes:

$$S_e = S = 3UI$$

Similarly to the single-phase case, the equivalent voltage and current may be separated into two components:

$$U_e^2 = U_{e1}^2 + U_{eH}^2 \,,\ I_e^2 = I_{e1}^2 + I_{eH}^2 \qquad [44]$$

where the index *1* marks the fundamental RMS components:

$$U_{e1}^2 = \frac{U_{a1}^2 + U_{b1}^2 + U_{c1}^2}{3} \,,\ I_{e1}^2 = \frac{I_{a1}^2 + I_{b1}^2 + I_{c1}^2}{3} \qquad [45]$$

and the index *H* marks the totalized non-fundamental RMS components:

$$U_{eH}^2 = \sum_{n=2}^{\infty} \frac{U_{an}^2 + U_{bn}^2 + U_{cn}^2}{3} \,,\ I_{eH}^2 = \sum_{n=2}^{\infty} \frac{I_{an}^2 + I_{bn}^2 + I_{cn}^2}{3} \qquad [46]$$

Also Equivalent Total Harmonic Distortion can be defined:

$$THD_{eU} = \frac{U_{eH}}{U_{e1}} \,,\ THD_{eI} = \frac{I_{eH}}{I_{e1}} \qquad [47]$$

The only apparent power definition known today, that holds this property for all the possible situations – balanced, unbalanced, sinusoidal or nonsinusoidal – has the mathematical expression suggested by F.Buchholz and explained by W.Goodhue[12].

Harmonics

When the load becomes nonlinear, the current waveform is not sinusoidal and contain many periodical components (harmonics).

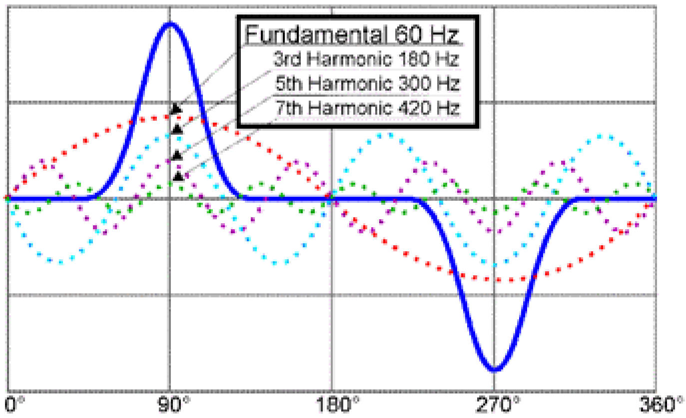
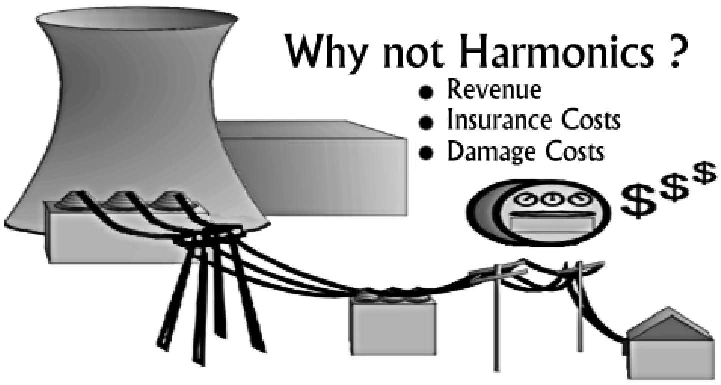


Fig.5: Demonstration of pulse current waveform

A harmonic is a multiple of the fundamental. For instance with general consumer power we use a line frequency in the USA of 60 Hertz. If we are talking about the third harmonic then that would be 60 x 3 or 180 Hertz. Harmonics are generated when the waveform is not sinusoidal or is distorted. One of the main culprits for generating harmonics is the switch-mode power supply. Switch-mode power supplies offer high efficiency, lightweight, in a small package, but they are rich in harmonics. The 3rd, 5th , 7th harmonics are all in phase at the peak of the fundamental and join to make the pulse waveform.

What is the problem with harmonics? You might ask. Well when it comes to power generation and consumption, if you really want to know what is going on, follow the money. When it comes to power, the electric (utility) company supply it, and we pay for it.

There are three good reasons the power/utility company wants to limit harmonics on the power mains:



1. **It affects how Utility companies bill you for the electricity you use.** Utility companies charge by the WATT but supply VA's. Lower power factor appliances and electric products that have a rectifier circuit at the main power input (switch-mode power supplies) draw current out of phase with the voltage. This effectively reduces the level of True Power ($UI\cos\varphi$), and the utility company doesn't get to charge (billing-wise) as much.
2. **All the power we use is derived from a three-phase source where the neutral current should be small.** When harmonics are introduced the three phases do not cancel and thus neutral lines end up carrying current. This means that the neutral lines need to carry a heavier load than normal, which could pose an electric fire hazard.
3. **Some smaller hydroelectric generators can even be damaged by harmonics.** The objective is to test the equipment under the conditions that will produce the maximum harmonic amplitudes under normal operating conditions for each harmonic component[W4].

When dealing with harmonics, the voltage distortion is generally the most important quantity. It is the voltage that will affect other connected loads. Resistive loads will consume more power due to the increased RMS value of the voltage, induction motors will become less efficient due to the counter rotating torque produced by negative sequence harmonic voltages, and the timing in digital circuits malfunction due to extra zero-crossings produced by severe voltage distortion.

The Benefits of Power Analysis

Power analysis is commonly used as a means for quality assurance production testing for a wide variety of electrical products. Power analysis, including harmonics and %THD, of a product is an excellent quality indicator of product performance, especially when compared to an identical model. For example, consider a medical device like an infusion pump generating excessive internal heat. A slightly binding rotor bearing or faulty power supply component may still allow the product to perform normally for awhile. With more subtle failure modes, just because the motor "runs" or the product appears normal, doesn't mean thermal runaway and product failure won't happen soon. A given product may be meeting its' basic specifications for operation yet consuming 10% more power than normal. Power analysis detects otherwise invisible secondary product problems so you can avoid more costly product field failures and improve your product's reliability reputation.

Measuring Instruments and methods

Until recently, the debate about P&E metering was confided, more or less, to a theoretical level. Only a few research-type instruments were available to correctly measure electrical quantities in the presence of harmonics using a variety of definitions. Now, however, the rapid progression of digital sampling technology in three-phase revenue metering has moved the issue from a purely research domain to an issue of practical application.
Traditional electromechanical or solid-state revenue meters were inherently limited to definitions that could be implemented in their technology. Consequently, they employed operating formulas which are accurate for sinusoidal waveforms, but lead to errors when currents and/or voltages are distorted. Many of the new digital sampling meters, which no longer have these technological limits, either simulate operating the principle of previous generation of electromechanical meters[6], or try to implement obsolete measuring concepts[11].

Power measurement

Wattmeters are used for the measurement of power. They may be either electromechanical (electrodynamic wattmeter) or electronic. They measure active power, but if suitably connected they may also be used for reactive power measurement.

Electromechanical wattmeters

The electrodynamometer movement is used extensively in measuring power. It may be used to indicate both DC and AC power for any waveform of voltage and current and it is not restricted to sinusoidal waveforms. The electrodynamometer used as a voltmeter or an ammeter has the fixed coil and the movable coil connected in series, thereby reacting to the effect of the current squared. When used as a single-phase power meter, the coils are connected in a different arrangement (see Fig.6)[8]. The electrodynamical movement has a deflection proportional to the average power.

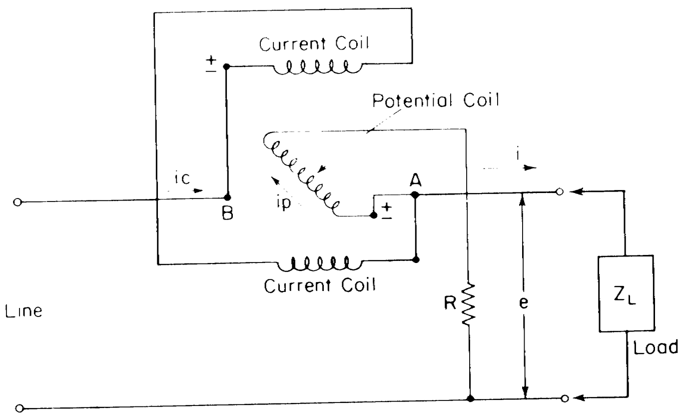


Fig.6: Diagram of an electrodynamometer wattmeter connected to measure the power of a single-phase load

Reactive power given by Eq.[6] for sinusoidal waveforms can be measured with wattmeters measuring the active power *P*, if the voltage on the wattmeter voltage coil is phase-shifted by 90 degree with respect to the voltage used when measuring the active power.

For the measurement of power in 3-phase systems, Blondel's theorem is of fundamental importance: The number of wattmeters required to measure correctly the active power of a multiphase system is by one less than the number of wires in the system[7].

Wattmeters based on electromechanical principle are not described in details in this document. The priority is given to the P&E measurement using digital approach.

Electronic wattmeters

Electronic wattmeters may be built around an analog multiplier or they use sampling methods. In the latter case, they often measure not only active power, but also Q, S, and the RMS values of $u(t)$ and $i(t)$, in which case they are called power analysers.
The input converters on the Fig.7 are used for the conversion of the load voltage and current to the voltage values corresponding to the input voltage ranges of the multiplier (VVC – voltage/voltage converter, CVC – current/voltage converter, M – multiplier, LPF – lowpass filter, DVM – digital voltmeter).

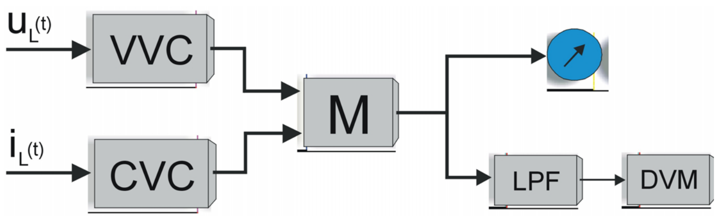


Fig.7: Block scheme of a single-phase electronic wattmeter

The presence of powerful nonlinear polluting loads (harmonics, nonsinusoidal, etc.) causes degradation of electric power quality, and to maintain necessary accuracy, the number of samples may have to be increased. As the sampling rate is related to the highest harmonic, the processing may become slow and require large computational time for higher order harmonics[11].

All of the algorithms for digital power measurement were originally developed for an analog system model. They perform the same computations but on discretized values of voltage and current signals, and thus represent digital aproximations of the analog processing schemes. For a digital system, direct application od digital signal processing seems more appropriate[11].

Energy measurement

Energy is measured by means of watt-hour meters, which are in fact simply integrating wattmeters. So circuits for the mesurement of power using wattmeters can also be used for measurement of energy consumption if wattmeters are replaced by watt-hour meters.

The energy meters that are being used by electricity suppliers in Europe can be devided into several types[14]:

- Anlog electromechanical meters – The operation of these meters is based on the induction principle of Ferraris. Several millions of these devices are installed in Europe.
- Purely digital meters – This type of meter samples the voltage and current signals and then calculates the energy quantity in a microprocessor.
- Mixed analog-digital meters

If the integrated power contains a non-zero DC part, only electronic or electrodynamic watt-hour meters can be used. (Electrodynamic watt-hour meters are integrating electrodynamic wattmeters, rather delicate instruments not much used any longer.) Common induction-type watt-hour meters cannot be used for integration of power containing a DC part, because that part of the power would not be measured (the angular frequency is a multiplication factor in the expression for their deflecting torque). Since they are frequency-dependent, they are also not suitable for measurement of the energy in circuits with strongly nonsinusoidal waveforms.
Polyphase watt-hour meters are used for the measurement of energy in 3-phase circuits. The number of meters inside a polyphase watt-hour meter must correspond to Blondel's theorem.

Special var-hour meters (instruments for the integration of reactive power) are also produced. They measure the “reactive energy“ W_Q, a quantity which has no physical meaning but which is used for finding the average power-factor of the load during a certain accounting period (for example one month). The average power-factor is computed as

$$(\cos\varphi)_{av} = \frac{W_P}{\sqrt{W_P^{\,2} + W_Q^2}}$$

where W_P is (active) energy and W_Q is reactive energy measured over the accounting period[7].

Induction Watt-hour meters

The induction watt-hour meter is not often found in a laboratory, but it is widely used for the commercial measurement of electrical energy. In fact, it is evident wherever a power company supplies the industrial or domestic consumer with electrical energy.
Fig.8 shows the element of a single-phase induction watt-hour meter in schematic form.

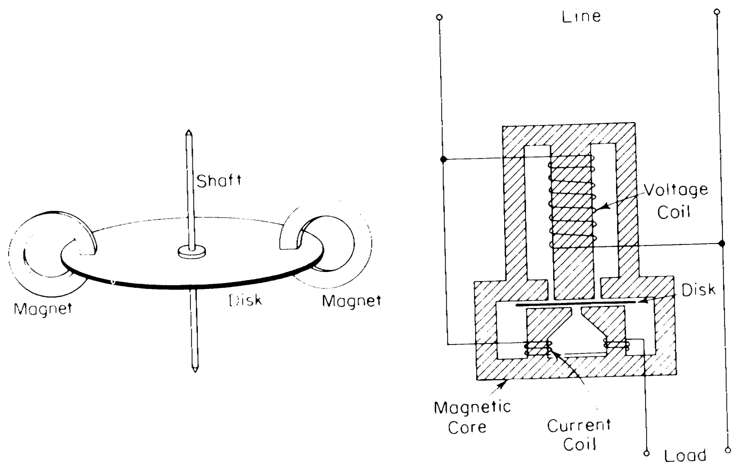


Fig.8: Element of a single-phase induction watt-hour meter

The current coil is connected in series with the line, and the voltage coil is connected across the line. Both coils are wound on a metal frame of special design, providing two magnetic circuits. A light aluminum disk is suspended in the air gap of the current-coil held, which causes eddy currents to flow in the disk. The reaction of the eddy currents and the field of the voltage coil create a torque (motor action) on the disk, causing it to rotate. The developed torque is proportional to the fieldstrength of the voltage coil and the eddy currents in the disk, which are in turn functions of the fieldstrength of the current coil. The number of rotations of the disk is therefore proportional to the energy consumed by the load in a certain time interval, and is measured in terms, of kilowatt-hours (kWh). The shaft that supports the aluminum disk is connected by a gear arrangement to the clock mechanism on the front of the meter, providing a decimally calibrated readout of the number of kWh[8].
The disadvantage of the induction watt-hour meter is its error if used in circuit with nonharmonic currents and voltages, caused by meters dependence no frequency. From this point of view, better instruments are electronic watt-hour meters[7].

Electronic Watt-hour meters

The conventional Ferraris induction disk metering based on electromechanical principles have been used worldwide since 19-th century. Recently, the requirements of precision measurement, data acquisition, energy management, and communication for modern electricity metering have stimulated the idea to develop intelligent meters based on electronic and microprocessor techniques[10].

The procedure for any electrical energy measurement is straightforward. Measure the current and voltage, then integrate this product over time.

An electronic meter needs to measure electrical parameters such as voltage, current, power, etc. Measurement of the true RMS value for instantaneous voltage and current using traditional analog methods become difficult if the input waveform is nonsinusoidal or distorted sinusoidal. The digital sampling method is a good alternative to solve this problem[10].

The electrical energy meter manufacturers have focused their research effort towards the development of modern and more precise energy meters for large customer, where the added precision justifies the necessary investment. As a result, electromechanical energy meters remain pervasive for residential application.
However, the advent of low cost microprocessors enables the development of a cost effective electronic energy meter for residential use as well[6].

Power Factor measurement

Since industrial application loads may require thousands of watts to operate, the resistance of the transmission lines supplying the power plays an important role. The lower power factor of the load, the greater the power that must be generated to supply the required value of the active power of the load. Because of this fact, special measures are taken to enhance the power-factor value of the load[7].

Phase-shift between a harmonic voltage *u(t)* and a harmonic current *i(t)* is usually expressed and measured as the power factor $\cos\varphi$ or true (total) power factor λ. Because the cosine function is positive both for negative $-90° < \varphi < 0°$ (RC combination circuit) and for positive $0° < \varphi < 90°$ (RL combination circuit), the sign of the difference cannot be found from the power-factor value. To indicate the sign of the phase shift, the power factor is characterized as leading or lagging according to the position of the current phasor with respect to the voltage phasor. So we speak about a lagging power factor for inductive (RL) load and about a leading power factor for capacitive (RC) load.
The power factor can be measured with special electromechanical pointer instruments called power-factor meters. These can measure the power factor of a one-phase load or of a three-phase load. They are based on a quadrature-coils meter movement (electrodynamometer or iron-vane)[7].

The indication of the pointer, which is connected to the movable element, is calibrated directly in terms of the phase angle or power factor.
Electromechanical power-factor meters are limited to measurement at comparatively low frequencies and are typically used at the powerline frequency. Phase measurement at higher frequencies often are more accurately and elegantly performed by special electronic instruments or techniques[8].

Harmonic analyzers

In the ideal case, application of a sinusoidal input signal to an electronic device, such as an amplifier, should result in the generation of a sinusoidal output waveform. Generally, however, the output waveform is not an exact replica of the input waveform because various types of distortion may arise. Distortion may be a result of the inherent nonlinear characteristics of the transistors in the circuit or of the circuit components themselves. Nonlinear behavior of circuit elements introduces harmonics of the fundamental frequency in the output waveform, and the resultant distortion is often referred to as harmonic distortion (HD).
A measure of the distortion represented by a particular harmonic is simply the ratio of the amplitude of the harmonic to that of the fundamental frequency, expressed as a percentage. Harmonic distortion is then represented by

$$HD_2 = \frac{B_2}{B_1},\ HD_3 = \frac{B_3}{B_1},\ HD_4 = \frac{B_4}{B_1}$$

where $HD_n(n = 2, 3, 4,...)$ represents the distortion of the n-th harmonic, B_n represents the amplitude of the n-th harmonic, and B_1 is the amplitude of the fundamental.
Several methods have been devised to measure the harmonic distortion caused either by a single harmonic or by the sum of all the harmonics[8].

Instruments which can measure power, true RMS, average, maximum and minimum values of voltage and current in the wide frequency band using the sampling method are produced under name of 'power analyzers'. They consist of a measurement part (U⇒U, I⇒U, S/H circuit, ADCs) and of a digital signal processor, which is used to compute the measured parameters of the sampled waveforms in the real time. The basic block diagram of power analyzer in shown in Fig.9.

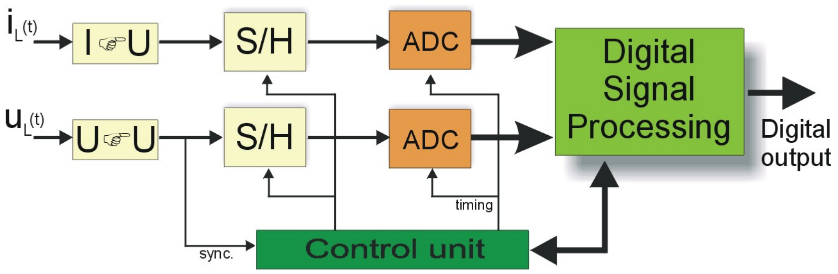


Fig.9: Basic block diagram of a power analyzer

The samples are processed directly in a special processor (real time processing). The rate of digital signal processing limits the usable sampling rate. For this reason more periods have to be sampled in this case[7].

Starting from the sampled data, different quantities are obtained, such as the current and voltage RMS values, their harmonic content, the active power, the harmonic active power, the power factor, the total harmonic distortion, and so on.

Spectrum and Harmonic Analyzers qualification

Instruments in the disturbance analyzer category have very limited harmonic analysis capabilities. Some of the more powerful analyzers have add on modules that can be used to Fast Fourier Transform (FFT) calculations to determine the lower order harmonics. However, any significant harmonic measurement requirements will require an instrument that is designed for spectrum analysis or harmonic analysis.
Important capabilities for useful harmonic measurements include:

- Capability to measure both voltage and current simultaneously so that harmonic power flow information can be obtained.
- Capability to measure both magnitude and phase angle of individual harmonic components (also needed for power flow calculations).
- Synchronization and a high enough sampling rate for accurate measurement of harmonic components up to at least the 37th harmonic (this requirement is a combination of a high sampling rate and a sampling interval based on the 60 Hz fundamental).
- Capability to characterize the statistical nature of harmonic distortion levels (harmonics levels change with changing load conditions and changing system conditions).

There are basically three categories of instruments to consider for harmonic analysis:

1. **Simple Meters.** It may sometimes be necessary to make a quick check of harmonic levels at a problem location. A simple, portable meter for this purpose is ideal. However, there are not many devices in this category available on the market. One microprocessor-based instrument calculates individual harmonics up to the 49th; as well as the RMS, the Total Harmonic Distortion (THD), and the Telephone Influence Factor (TIF). The telephone influence factor is similar to the THD except it

is weighted based on the sensitivity of the human ear to provide a better measure of the potential for interference to voice communication circuits. The simplest instrument uses an analogue circuit to filter out the 60 Hz for calculation of the THD.

2. **General Purpose Spectrum Analyzers.** Instruments in this category are designed to perform spectrum analysis on waveforms for a wide variety of applications. They are general signal analysis instruments. The advantage of these instruments is very powerful capabilities for a reasonable price since they are designed for a broader market than just power system applications. The disadvantage is that they are not designed specifically for sampling 60 Hz waveforms and, therefore, must be used carefully to assure accurate harmonic analysis. There are a wide variety of instruments in this category.
3. **Special Purpose Power System Harmonic** Analyzers. Besides the general purpose spectrum analyzers described above, there are also a number of instruments and devices which have been designed specifically for power system harmonic analysis (Fig.10). These are based on the FFT with sampling rates specifically designed for determining harmonic components in power signals. They can generally be left in the field and include communications capability for remote monitoring.

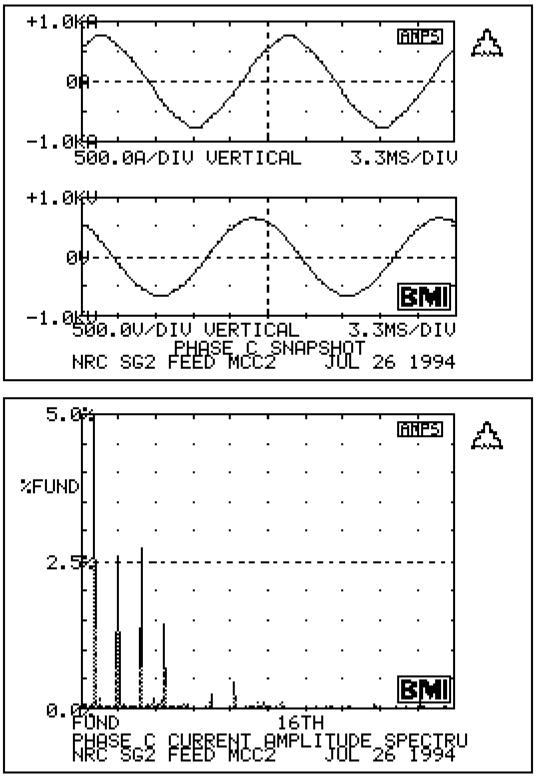


Fig.10: Harmonic Analyzer output

A few minimal requirements for a harmonics analyzer follow:

- Simultaneous measurement of voltage and current so that harmonic power flow can be obtained.
- Sampling of the waveform synchronized to the fundamental frequency, to insure accurate calculation of harmonic phase angles.
- A sampling rate sufficient to determine up to the 50th harmonic or better.
- High resolution analog to digital conversion. This is necessary because high order voltage and current harmonics are typically several orders of magnitude less than the full-scale reading of the instrument.

Harmonic distortion is a continuous phenomena. It can be characterized at a point in time by the frequency spectrums of the voltages and currents. However, for proper representation, measurements over a period of

time must be made and the statistical characteristics of the harmonic components and the total distortion determined.
Several standards or recommended practices are discussed in great detail in the "Power Quality Standards and Planning Limits" section. These standards deal with measurement techniques, application guidelines, and design guidelines. The standards that pertain to harmonics will be briefly reviewed.

Combination Disturbance and Harmonic Analyzers

The most recent instruments combine limited harmonic sampling and energy monitoring functions with complete disturbance monitoring functions as well (Fig.11). The output is graphically based and the data is remotely gathered over phone lines into a central database. Statistical analysis can then be performed on the data. The data is also available for input and manipulation into other programs such as spreadsheets and other graphical output processors[W7].

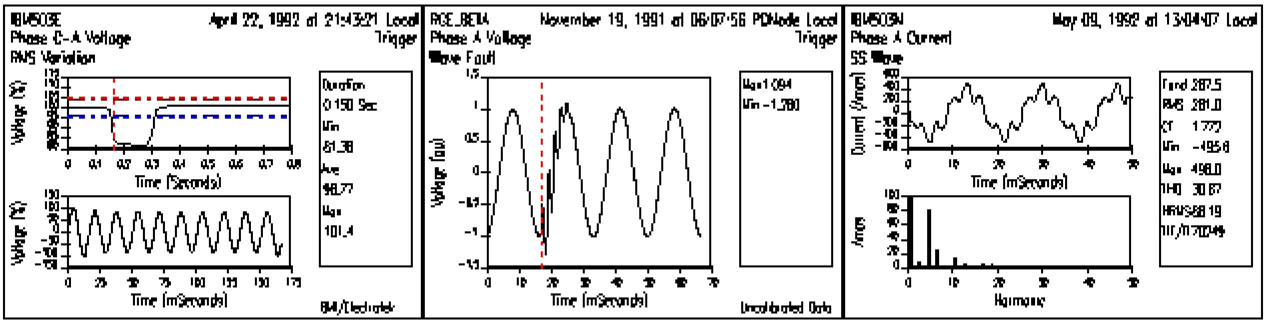


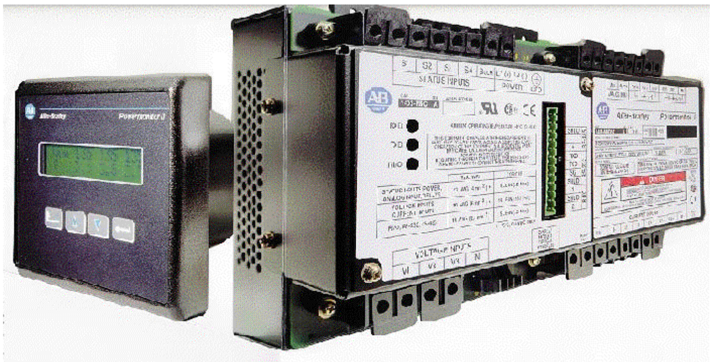
Fig.11: Output from Combination Disturbance and Harmonic Analyzer

Present state of P&E measurement

Evaluation engineers are being challenged to provide accurate, detailed descriptions of the power consumption profile of new line-powered electrical devices. Due to the continued emphasis on efficiency, electrical designers are becoming increasingly dependent on semiconductors to provide precise control of fine current, thereby obtaining the lowest possible power consumption. As a result, the traditional tools and methods for measuring power no longer provide accurate or complete results. Electrical loads, which have semiconductors in the line circuits include everything from large motor drives to fluorescent lighting ballasts. To properly evaluate one of these devices, you need a line-voltage power meter.
Power meters must measure and record power in kilowatts, the true power factor (λ), the displacement power factor (DPF), true-rms current, total harmonic distortion (THD), and the harmonic current spectrum up to at least the 31st harmonic.
Harmonic measurements are particularly important if the device being evaluated will be sold in European countries where local governments place strict limits on harmonic currents generated by loads connected to the power line. Traditional power meters used analog multipliers or Hall-effect devices to accurately measure power consumption, but the analog circuits were unable to measure harmonic distortion. Modern power meters typically use a sampling circuit in conjunction with a microprocessor to provide both the power consumption and the harmonic information. Good accuracy for power measurements can be achieved with a rate of about 100 samples per line cyc1e. A bandwidth of at least 2 kHz is required to measure harmonics up to the 31-st. Power meters should include the capability to accurately measure all the parameters mentioned plus a few added features to make the measuring job safer and easier. Some power meters have enhanced memory capabilities and a record mode that takes a comprehensive snapshot of data. A serial interface makes it possible to download information to a PC. Application software allows data to be viewed on screen in graphics and tables. Data and graphics can be easily transferred to a spreadsheet or word processor for custom reports or saved to a disk file for documentation and future use[16].

Examples of power monitoring products

Rockwell Automation/Allen-Bradley developed power monitoring device called PowerMonitor (Bulletin 1400) and the newest version PowerMonitor II. (Bulletin 1403). The PowerMonitor is microprocessor based monitoring and control device well suited for variety of electric power applications. The PowerMonitor manage 3-phase industrial, commencial, and utility power systems[W5].



Main PowerMonitor's featurtes:

Analyze and capture:

- kW, kWH, kVA, kVAR
- Demand metering
- Power factor
- Line frequency
- 4 voltages
- 4 currents

Archival (Data logging capabilities data and time stamp when disturbances are occurring):

- Min/Max log
- Event log
- Snapshot log

- Waveform capture log

Outputs/Communications:

- Three sets of SPDT (form C) contacts, user configured
- Analog output, user configured
- Four status inputs
- Simultaneously communicates on Allen-Bradley Remote I/O and RS-232C or RS-485.

Continuous, high-speed data sampling at 10.8 kHz yields 180 samples/cycle @ 60 Hz under ALL operating conditions.

High accuracy for both: Standard metering parameters (< 0.05% F.S.) and Harmonic parameters (< 5% @ 41st harmonic)

Summit Technology, Inc. offers another instrument for the measurement and analysis of electric power – The PowerSight. This product provides capabilities for demand analysis, harmonic analysis, power quality analysis and datalogging[W6]. Frequency response through the 50th harmonic allows for the accurate measurement of harmonic distortion.

Main features:

- 12 voltage measurements,
- 16 current measurements
- True RMS and VA power
- Monitor demand, harmonics, or disturbances
- Serial communications port to upload waveforms, data log and remote control
- PC Analysis software to help catalog, and generate reports

Power Measurement Company is the world's leading manufacturer of advanced, multi-function digital power meters and billing meters. They provide solutions for:

Revenue metering	Power quality monitoring
Power & energy metering	Power outage avoidance
Load aggregation	Equipment protection
Cost allocation	Distribution automation
Demand management	Power Factor control

The 7500 ION is a high-visibility power quality and energy meter suitable for monitoring key distribution points and loads sensitive to power disturbances. It features:

- Large graphical display screen
- High-accuracy measurements
- Power quality analysis
- Historical trending
- ½-cycle setpoint response
- Additional neutral voltage and ground current inputs

It's flexible on-board communications mean simple integration and low installation costs:

- Ethernet port (optional) includes EtherGate capability (provides network access for up to 31 additional meters on an RS-485 network.)
- Modem port (optional) includes
- ModemGate (available 99Q4).
- Two serial ports (one RS-232/485 and one RS-485).
- One infrared port.
- Multiple protocol support: Modbus RTU, DNP 3.0, ION.

P&E measurement in Electric Utilities

Very interesting survey was done by The IEEE Working group on nonsinusoidal situations[21] in 1995. 50 utilities in the USA and Canada responded to a questionnaire survey regarding concerns about nonsinusoidal waveforms in the electric distribution network.
There was little agreement among responding utilities on a unique definition for three-phase apparent power. Only 52% of the responding utilities gave a detailed, preferred definition of apparent power S, and some utilities referred to more than one definition.
22% reported using the Prasor VA

$$S = \sqrt{(P_a + P_b + P_c)^2 + (Q_a + Q_b + Q_c)^2}$$

and another 22% reported using the Arithmetic VA

$$S = S_a + S_b + S_c$$

where, given P_{abc} and Q_{abc} as the active and reactive powers on phases A, B, and C

$$S_a^2 = P_a^2 + Q_a^2,\ S_b^2 = P_b^2 + Q_b^2,\ S_c^2 = P_c^2 + Q_c^2$$

6% reported using instrumentation based on the definition

$$S = 1.11^2 U_{avg} I_{avg}$$

where U_{avg} and I_{avg} are full-wave rectified average values.

Apparently, very few of the responding utilities are concerned yet with the problem of power factor measurement and correction in the three-phase systems with unbalanced loads, because only 4% claimed to use the definition

$$S_e = 3U_e I_e$$

where $U_e = \sqrt{\dfrac{U_a^2 + U_b^2 + U_c^2}{3}}$ and $I_e = \sqrt{\dfrac{I_a^2 + I_b^2 + I_c^2}{3}}$

where U_{abc} and I_{abc} are the RMS values of the line voltages and currents.

Of the utilities that reported measuring apparent power, 12% reported using digitally-sampled microprocessor-based measurements, 12% reported using thermal demand measurements, and 4% reported using rectithermal measurements. The remainder did not report their measuring techniques.

All utilities reported monitoring reactive power *Q* for large customers. KWh-meters with phase-shifting transformers are the most common measuring devices. In certain applications, *Q* is computed from *S* and *P*. Of the utilities that use kvar or kvarh metering, 48% employ phase shift transformers and displaced voltage; 16% employ digital sampling and microprocessors; 12% employ time-division multipliers with electronic phase-shifters; 8% employ auto-transformers and capacitor-resistor phase shift; 6% employ solid-state kvar meters of an undetermined algorithm; 6% employ kvar demand meters of an undetermined algorithm, and 2% report using kvar transducers of an undetermined algorithm.

Microprocessor-based measurement

To get all wanted information about the power flow, the distribution of the harmonics in a system, the possible changing of fundamental amplitude and supply frequency or phase shifting, only a measuring of the three phase voltages and currents is needed.
To measure all harmonics up to, for example, 19-th harmonic a measuring of the fundamental wave by 250 points per period is necessary. That means that 19-th harmonic is still measured by 13 measuring points per its period. Measuring a 50Hz system a distance of 80µs between two measuring points is needed.

The continuous measured waveform is replaced by a succession of discrete values. If integration is used in mathematical definition, it is replaced by summation. In order to convert the continuous waveform of voltage to a succession of voltage samples, a sampler and a fast ADC (Analog-to-Digital Converter) have to be used[7].

Power is measured by multiplying digitized samples of the current and voltage waveforms.
The average power measured over an interval *T* is given by [3]

$$P = \frac{1}{T}\int_0^T u(t)\cdot i(t)dt$$

This integral can be approximated by the summation

$$P = \frac{1}{N}\sum_{n=0}^{N-1} u(n)i(n)$$

where *u(n)* and *i(n)* are simultaneous samples of voltage and current equally spaced in time, and *N* is a number of samples taken over a period *T*. The samples need not be taken over a single period but can be spread over *m* periods (where *m* is an integer) allowing the sampling interval to be increased accordingly[17].
The conditions required:

1) the sampling must occur over an integer number of waveform; and
2) the waveform is stationary for the duration of measurement.

While the measurement of greatest interest is power, other function can be also derived from the raw data.
The True RMS value of voltage and current is defined by the relation

$$U = \sqrt{\frac{1}{T}\int_0^T u^2(t)dt}\,,\ I = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$$

The integral for the whole period can be substituted by the summation

$$U = \sqrt{\frac{1}{N}\sum_{n=0}^{N-1} u^2(n)}\,,\ I = \sqrt{\frac{1}{N}\sum_{n=0}^{N-1} i^2(n)}$$

A watt-hour measurement capability is also provided using a clock, which is started when the watt-hour function is selected. Whenever data is processed the clock is read and the time since the last measurement update is computed. This difference is multiplied by the latest power measurement, and the result added to the accumulated watt-hour measurement. This approach assumes that the power remains essentially constant between updates[17].

Common configuration

The basic configuration of the digital sampling wattmeter is shown in Fig.12. The instrument is controlled by single-board computer. Signal conditioning board (interface) comprises a selectable gain voltage amplifiers and current-to-voltage converters (sensors) for the voltage and current signals, respectively.

The digitization is performed by AD converters which are simultaneously triggered at a sampling interval previously determined by processor.
The timing information from which the sampling period is determined is derived from the zero crossing of the voltage signal.

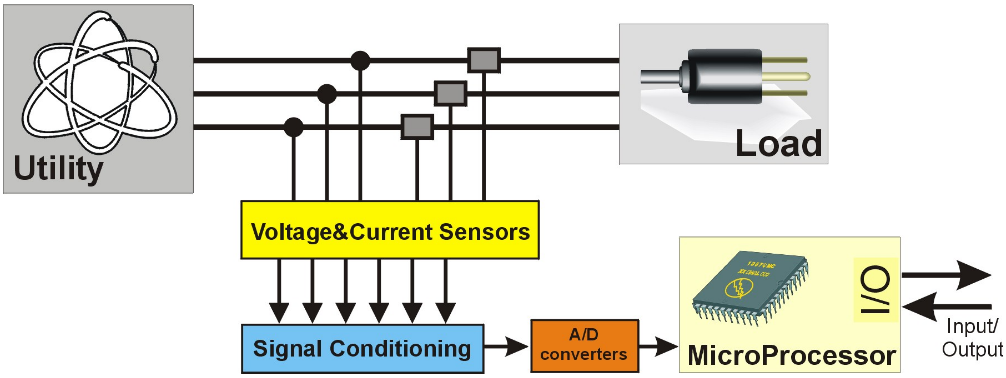


Fig.12: Basic configuration of the digital sampling wattmeter

In the last ten years, one of the most meaningful advances in electronics in the digital signal processor (DSP). This single-chip device is a high performance microprocessor with internal hardware specially designed to execute quickly successive multiply and accumulate operation. The DSP is thus suitable to implement digital filters, FFT, speed or image processing, or more complex data processing algorithm. The use of DSP's exhibits indisputable advantages and benefits also for measurement and control systems[18].

Current measurement can be done by measuring the voltage drop across a resistive shunt, or by current transformer, or by Hall-effect sensor. Voltage can be measured by means of voltage divider, voltage transformer or Hall-effect sensor.
Values of the signals sometimes have to be conditioned into a format suitable for an AD converter. Additionally, the load circuit should be isolated from the measurement circuit.

Application of Fourier transform

In the case of periodic signals, the Fourier series expansion can be used to perform spectral analysis of the voltage and current waveforms. If digital signal processing is used, then the discrete Fourier Transform (DFT) is usually applied to the samples of measured signals.
Since a digital computer can only store and manipulate a finite set of numbers, it is necessary to represent an analogue signal by a finite set of values. The first step in doing so is to sample the analogue signal *u(t)* to obtain a discrete sequence *u(n)*. Without loss of generality, we can assume that these samples are defined for n in the range [0, *N*-1]. We can take the discrete-time Fourier transform of the sequence as

$$U(\Omega)=\sum_{n=1}^{N-1} u(n)\exp[-j\Omega n]$$

This is still not in a form suitable for machine computation, since Ω is a continuous variable taking values in [0, 2π]. The final step, therefore, is to evaluate $U(\Omega)$ at only a finite number of values, Ω_k, by a process of sampling uniformly in the range [0, 2π] as

$$U(\Omega_k)=\sum_{n=1}^{N-1} u(n)\exp[-j\Omega_k n] \text{ , k=0, 1, … , M-1}$$

where $\Omega_k = \frac{2\pi}{M} k$.

The number of frequency samples, *M*, can be any value. However, we choose it to be the same as the number of time samples, *N*. With this modification, and writing $U(\Omega_k)$ as *U(k)*, we finally have

$$U(k)=\sum_{n=1}^{N-1}u(n)\exp\left[-j\frac{2\pi}{N}nk\right]$$

One of the reason of the widespread use of the DFT and other discrete transforms is the existence of algorithms for their fast and efficient computation on a computer. For the DFT, these algorithms collectively goes under the name of Fast Fourier Transform (FFT) algorithm. The basic idea is to divide the given sequence into subsequences of smaller length. We then combine these smaller DFTs suitably to obtain the DFT of the original sequence.
If the sampling period is f_s, the measured spectrum is displayed for the frequencies from 0 to $f_s/2$ (the frequency band of the analyzer).

Digital processing allows the instrument to make quantitative measurements of a variety of parameters, for the impact of harmonics on the power system evaluation:

- the harmonic voltages U_v and currents I_v content;
- the voltage and current RMS (effective) values:

$$U_{RMS}=\sqrt{\sum_{v=0}^{N}U_v^2}\ ,\ I_{RMS}=\sqrt{\sum_{v=0}^{N}I_v^2}$$

- their crest (ratio of peak to RMS) factor:

$$CF_U=\frac{U_{peak}}{U_{RMS}}\ ,\ CF_I=\frac{I_{peak}}{I_{RMS}}$$

- the active power:

$$P=3\sum_{v=0}^{N}U_v I_v\cos\varphi_v$$

where φ_v is the phase angle for sinusoidal excitation of v-th harmonic frequency;

- the harmonic active power;
- the apparent power

$$S=3U_{RMS}I_{RMS}$$

- the power factor, defined as the ration if the active power to the apparent power

$$PF=\frac{P}{S}=\frac{\sum_{v=0}^{N}U_v I_v\cos\varphi_v}{U_{RMS}I_{RMS}}=\frac{\sum_{v=0}^{N}U_v I_v\cos\varphi_v}{\sqrt{\sum_{v=0}^{N}U_v^2}\sqrt{\sum_{v=0}^{N}I_v^2}}$$

- the total harmonic distortion (*THD*) defined as the ratio of the RMS value of the harmonics to the RMS value of the original distorted signal:

$$THD_U=\frac{\sqrt{\sum_{v=2}^{N}U_v^2}}{U_1}\ ,\ THD_I=\frac{\sqrt{\sum_{v=2}^{N}I_v^2}}{I_1}$$

- the fundamental frequency f_1.

In the algorithm proposed in refer.[18] the harmonic measurement is carried out by assuming the frequency f_1 of the fundamental component of the input signal, known. The sampling frequency f_s is expressed by the notation $f_s=2(N+1)f_1$. As a consequence f_1 must be known to set sampling frequency.
Unfortunately this frequency may change while the system is in operation and then should be measured. This is a problematic task in some applications because of the noise superimposed on the signal, which produces several zero crossing in a period. To solve this problem a digital filter, with a cutoff frequency variable with the input signal characteristics, can be implemented[18].

For the estimation of the effects produced on the communication line, the susceptibility of the circuit to the effects of induced interference can be considered. Because the produced effects are not uniform over the audio-frequency spectrum, they are estimated at the maximum ear sensitivity, at about 800Hz, as required by International Consultation Commission on Telephone and Telegraph Systems (CCITT). The response is psophometrically weighted to obtain the standard parameters (for example equivalent disturbing voltage (EDV) is a voltage at 800Hz which, if applied to the power line, would cause the same interfering effect, to be experienced in a nearby telephone line, as does the voltage on the power line and its harmonics.

Modeling & Simulation

Numeric simulation was employed to demonstrate different features and results of the computation methods and algorithms mentioned in the previous chapters. Software package Matlab/Simulink was used for this purpose.

Models of power system

In order to evaluate various measuring systems under both sinusoidal/nonsinusoidal and balanced/unbalanced conditions, three different loads supplied from 3-phase source were modeled:

1• 3-phase Induction motor (Fig.13) – connected directly to the grid. Induction motor represents 3-phase sinusoidal and balanced load.

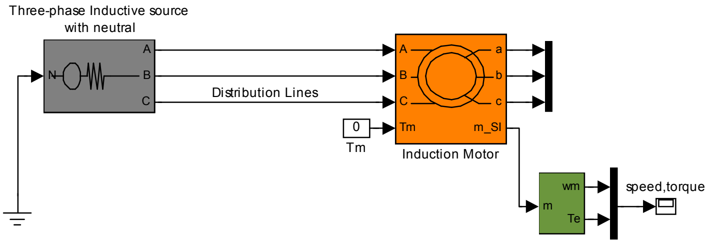


Fig.13: Simulink model of a sinusoidal and balanced load

2• 3-phase Bridge Diode rectifier (Fig.14) – represents 3-phase nonsinusoidal balanced load.

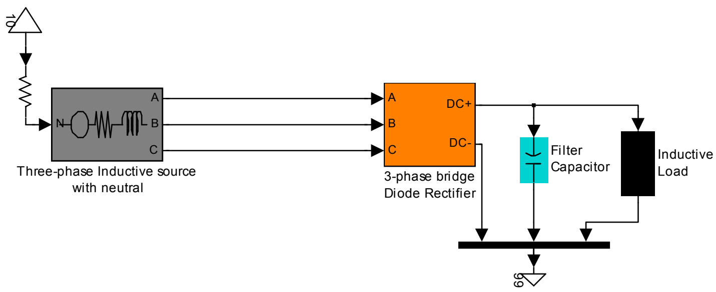


Fig.14: Simulink model of a nonsinusoidal and balanced load

3• Single-phase Diode rectifier (Fig.15) – represents nonsinusoidal and unbalanced load.

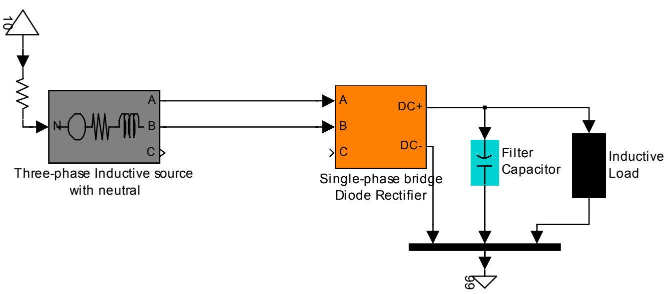


Fig.15: Simulink model of a nonsinusoidal and unbalanced load

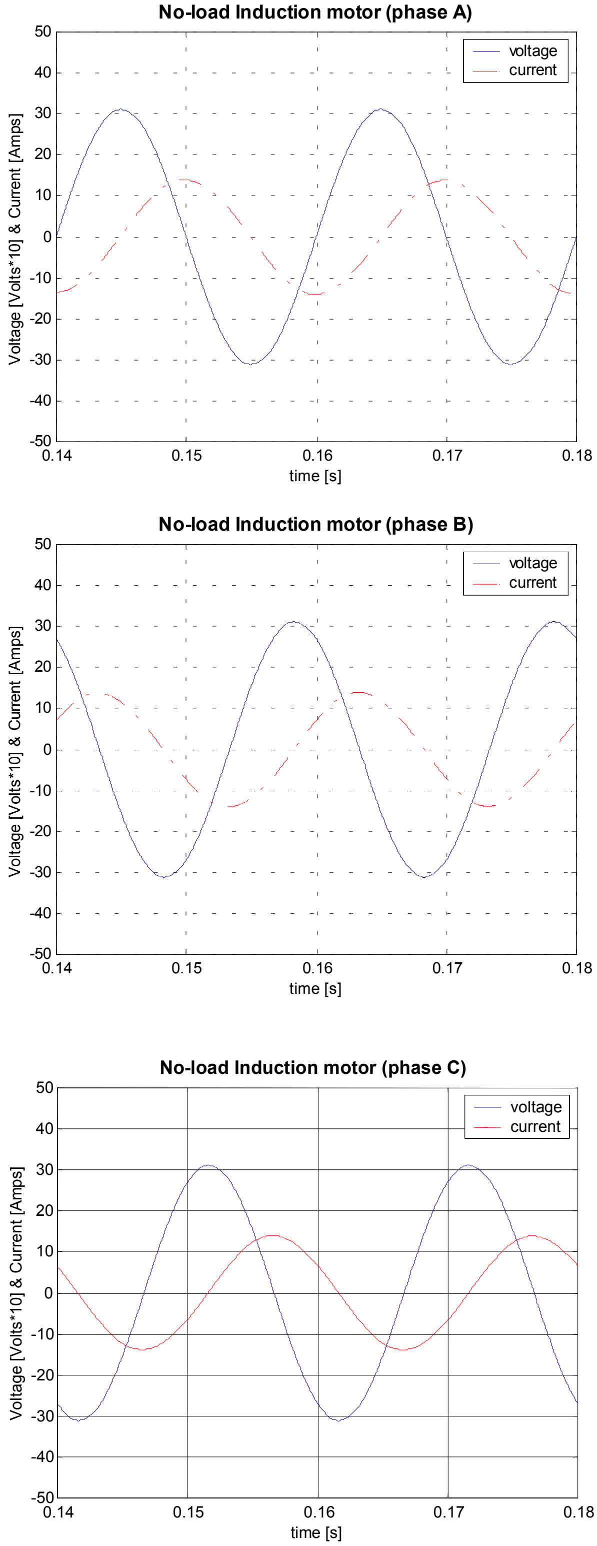


Fig.16: Steady-state waveforms of Induction motor Voltages and Currents

Voltage & Current waveforms

Currents and voltages are measured on the load terminals. Measured waveforms in a steady-state are shown in Fig.16, 17, and 18 for different types of loads. As you can see in the Fig.16, both currents and voltages drawn by 3-phase Inductive motor are sine-shaped for all phases. Current is phase-shifted with respect to voltage, because Induction motor is a typical inductive load.

On the other hand, currents and voltages drawn by a rectifier (3-phase or single-phase) are no more sinusoidal. Fig.17 shows the waveforms measured at the input of a three-phase rectifier. Due to nonlinearity of diodes used in the rectifier to convert AC energy to a DC form the current has nonsinusoidal shape and consists of many harmonics (Fig.20). The voltage is also influenced by nonlinearity of the load, but a harmonic contents is not so high in this case (Fig.19).
In the Fig.21 are shown fundamental voltage and current in the phase A and the Fig.22 demonstrates a contents of harmonics of drawn current (only harmonics with a dominant magnitude are concerned).

Regards to harmonics, the situation is getting worse for a single-phase rectifier. In addition, this load is nonsymetrical and therefore drawn currents are not the same in all phases (Fig.18). Fig.23, 24, 25, and 26 describe the same features as the figures for 3-phase rectifier.

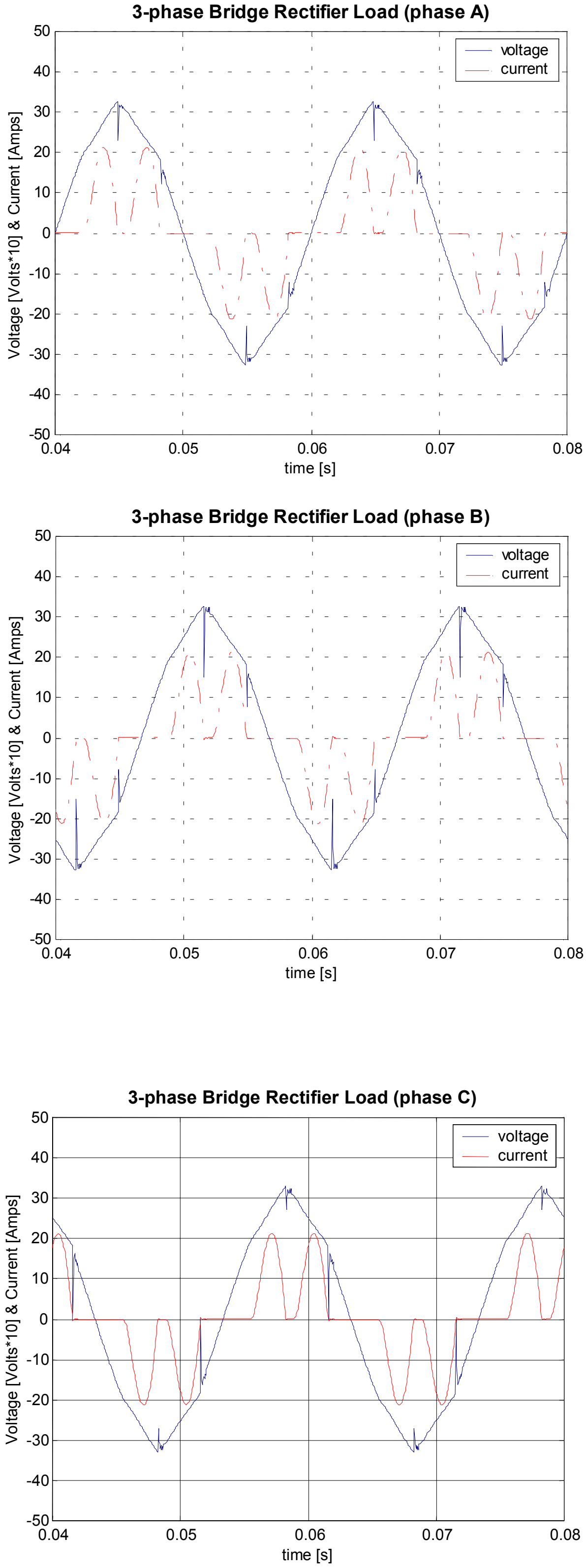


Fig.17: Steady-state waveforms of 3-phase rectifier Voltages & Currents

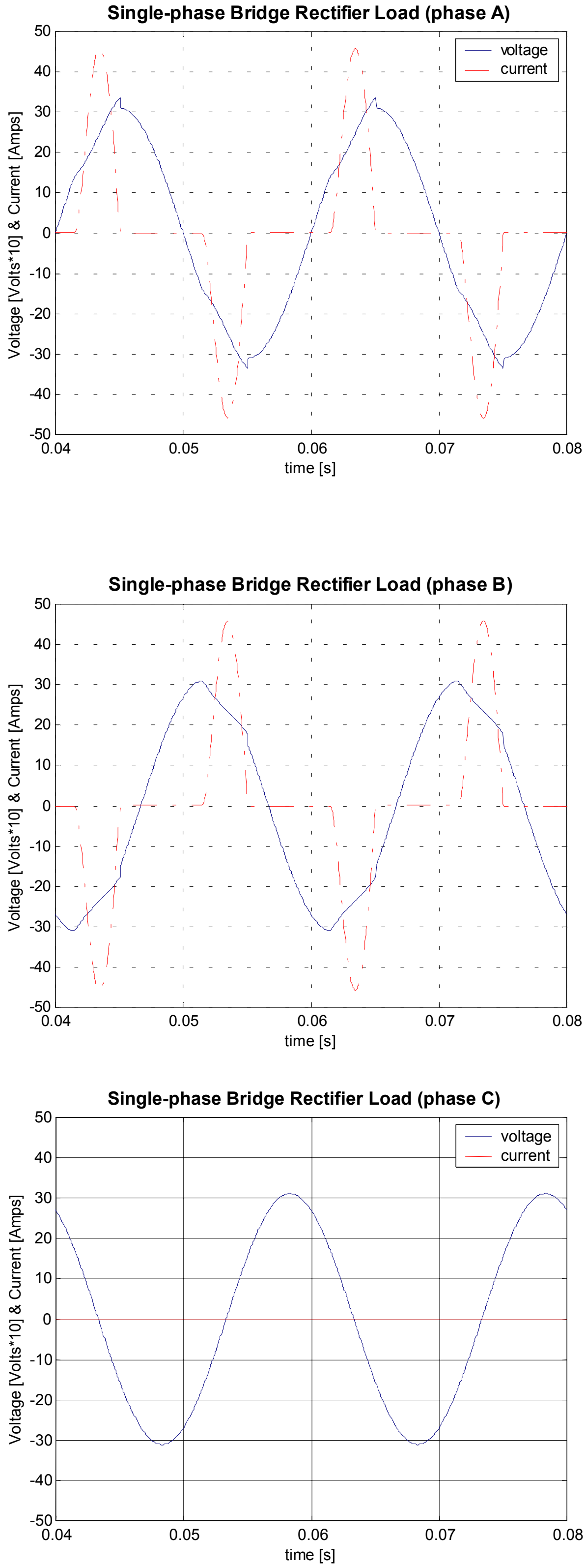


Fig.18: Steady-state waveforms of Single-phase rectifier Voltages & Currents

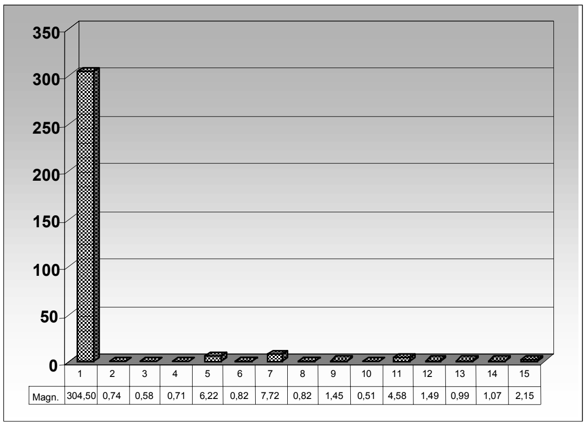


Fig.19: Harmonic spectrum for a 50Hz voltage of 3-phase rectifier

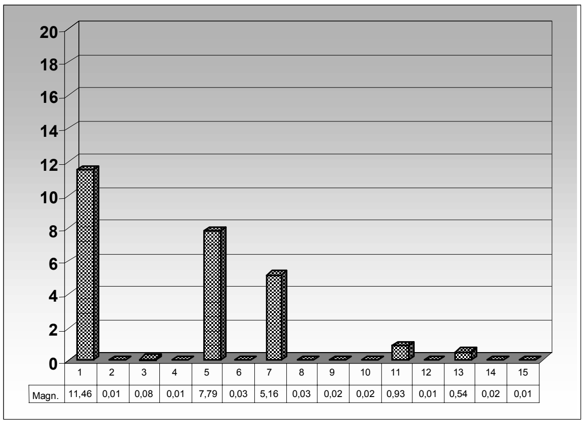


Fig.20: Harmonic spectrum for a 50Hz current of 3-phase rectifier

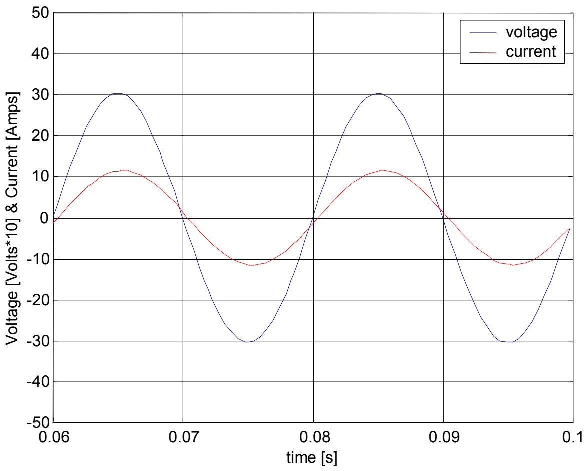


Fig.21: Fundamental Waveforms of 3-phase rectifier

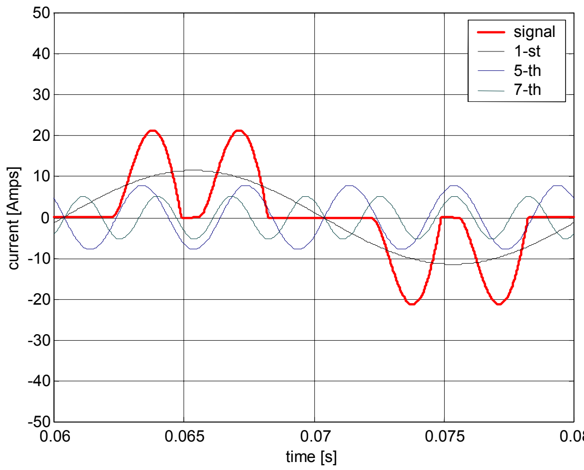


Fig.22: Harmonic contents of line current for 3-phase rectifier

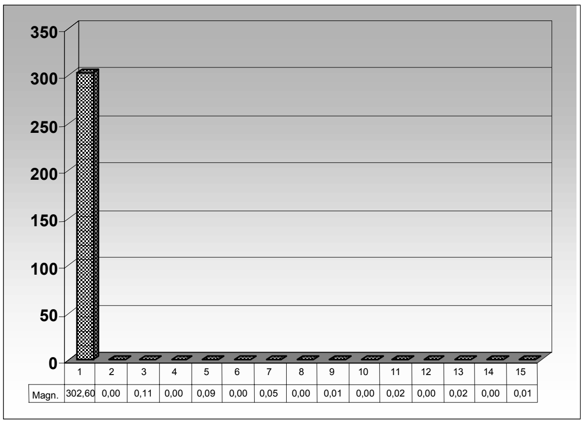


Fig.23: Harmonic spectrum for a 50Hz voltage of single-phase rectifier

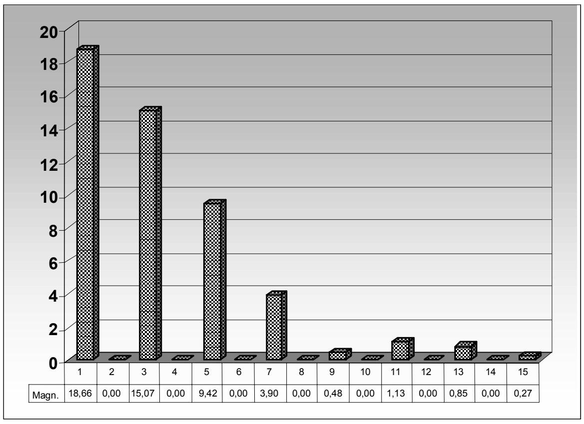


Fig.24: Harmonic spectrum for a 50Hz current of single-phase rectifier

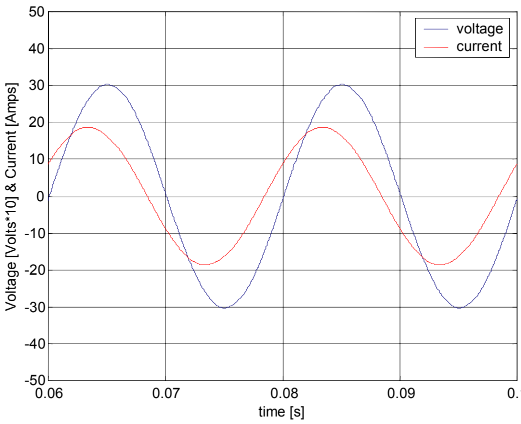


Fig.25: Fundamental Waveforms of single-phase rectifier

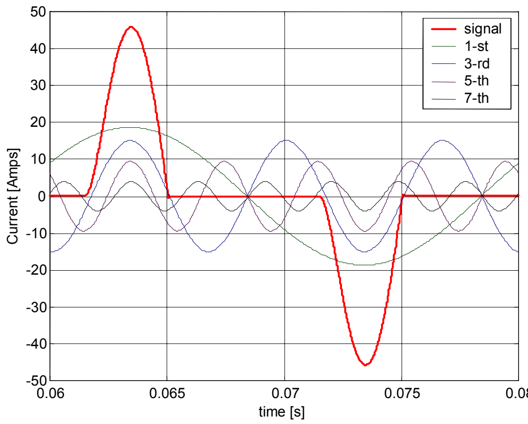


Fig.26: Harmonic contents of line current for Single-phase rectifier

Simulation results

Several methods were applied to get the value of quantities describing power and energy consumption. In this chapter the results of simulations are sorted into the tables to demonstrate differences between these methods. The equations and definitions used for calculation of the quantities are also included (details are in the Chapter Fundamentals of P&E measurement).
The Simulink model used for simulation of power system is shown in Fig.27. Currents and voltages are “measured” in the input of the load (Induction motor, 3-phase rectifier, of Single-phase rectifier). In this case, 3-phase 3-wire system is assumed.

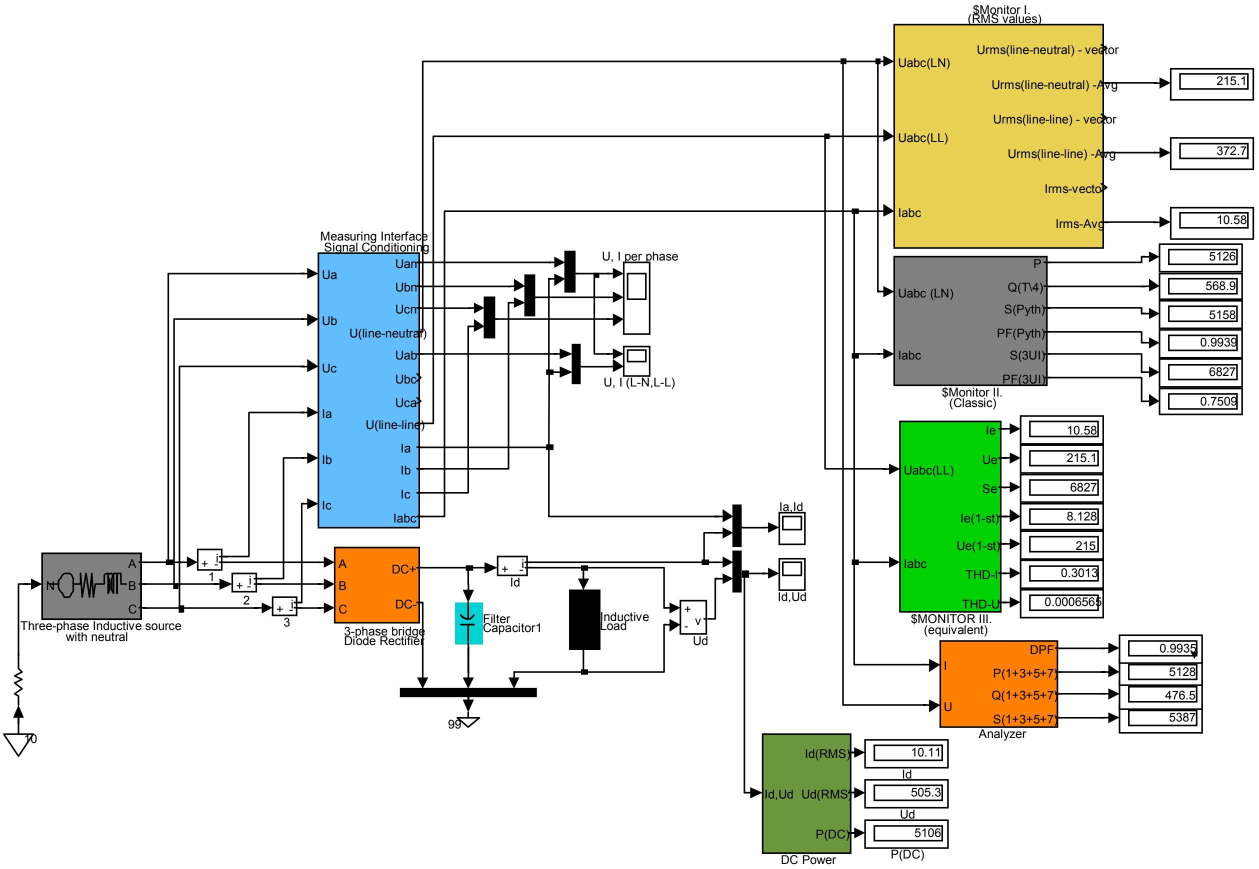


Fig.27: Simulink model for the power consumption evaluation

Voltage & Current measurement

Line-to-neutral RMS voltage (per phase)

$$U_{aN} = \sqrt{\frac{1}{T}\int_0^T u^2{}_{aN}(t)dt}\ ,\ U_{bN} = \sqrt{\frac{1}{T}\int_0^T u^2{}_{bN}(t)dt}\ ,\ U_{cN} = \sqrt{\frac{1}{T}\int_0^T u^2{}_{cN}(t)dt}$$

Line-to-line RMS voltage (per phase)

$$U_{ab} = \sqrt{\frac{1}{T}\int_0^T u^2{}_{ab}(t)dt}\ ,\ U_{bc} = \sqrt{\frac{1}{T}\int_0^T u^2{}_{bc}(t)dt}\ ,\ U_{ca} = \sqrt{\frac{1}{T}\int_0^T u^2{}_{ca}(t)dt}$$

Average line-to-neutral and line-to-line RMS voltages

$$U_{LN(avg)} = \frac{U_{aN} + U_{bN} + U_{cN}}{3}\ ,\ U_{LL(avg)} = \frac{U_{ab} + U_{bc} + U_{ca}}{3}$$

Equivalent RMS voltage

$$U_e = \sqrt{\frac{U_{aN}^2 + U_{bN}^2 + U_{cN}^2}{3}} = \sqrt{\frac{U_{ab}^2 + U_{bc}^2 + U_{ca}^2}{9}}$$

Output RMS voltage of a rectifier (not present for Induction motor)

$$U_{DC} = \sqrt{\frac{1}{T}\int_0^T u^2{}_{DC}(t)dt}$$

RMS Voltages [Volts]			
Voltage	Induction Motor	3-phase Rectifier	Single-phase Rectifier
U_{aN}	219.8	215.2	214.4
U_{bN}	219.8	215.1	212.5
U_{cN}	219.8	215.1	220.0
$U_{LN(avg)}$	219.8	215.1	215.6
U_{ab}	380.7	372.7	365.8
U_{bc}	380.7	372.6	375.7
U_{ca}	380.7	372.7	378.9
$U_{LL(avg)}$	380.7	372.7	373.5
U_e	219.8	215.0	215.4
U_{DC}	none	505.3	479.0

Line RMS current (per phase)

$$I_a = \sqrt{\frac{1}{T}\int_0^T i^2{}_a(t)dt}\ ,\ I_b = \sqrt{\frac{1}{T}\int_0^T i^2{}_b(t)dt}\ ,\ I_c = \sqrt{\frac{1}{T}\int_0^T i^2{}_c(t)dt}$$

Average Line RMS current

$$I_{(avg)} = \frac{I_a + I_b + I_c}{3}$$

Equivalent RMS current

$$I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2}{3}}$$

Output DC current of a rectifier (not present for Induction motor)

$$I_{DC} = \sqrt{\frac{1}{T}\int_0^T i^2{}_{DC}(t)dt}$$

RMS Currents [Volts]			
Current	Induction Motor	3-phase Rectifier	Single-phase Rectifier
I_a	9.82	10.58	18.48
I_b	9.82	10.58	18.48
I_c	9.82	10.58	0
I_{avg}	9.82	10.58	12.32
I_e	9.82	10.58	15.09
I_{DC}	none	10.11	9.5

Harmonic Analysis

The Fast Fourier Transform (FFT) was used for a harmonic analysis of voltages and currents up to the 15-th harmonic.

Harmonic Analysis of Voltage & Current (phase A)						
Harmonic	Induction Motor		3-phase Rectifier		Single-phase Rectifier	
Magnitude	U_{aN} [V]	I_a [A]	U_{aN} [V]	I_a [A]	U_{aN} [V]	I_a [A]
DC	0,00	0,00	0,00	0,01	0,00	0,00
1-st	311,10	13,88	302,60	11,46	304,50	18,66
2-nd	0,00	0,00	0,00	0,01	0,74	0,00
3-rd	0,00	0,00	0,11	0,08	0,58	15,07
4-th	0,00	0,00	0,00	0,01	0,71	0,00
5-th	0,00	0,00	0,09	7,79	6,22	9,42
6-th	0,00	0,00	0,00	0,03	0,82	0,00
7-th	0,00	0,00	0,05	5,16	7,72	3,90
8-th	0,00	0,00	0,00	0,03	0,82	0,00
9-th	0,00	0,00	0,01	0,02	1,45	0,48
10-th	0,00	0,00	0,00	0,02	0,51	0,00
11-th	0,00	0,00	0,02	0,93	4,58	1,13
12-th	0,00	0,00	0,00	0,01	1,49	0,00
13-th	0,00	0,00	0,02	0,54	0,99	0,85
14-th	0,00	0,00	0,00	0,02	1,07	0,00
15-th	0,00	0,00	0,01	0,01	2,15	0,27

Total Harmonic Distortion (per phase)

$$THD_{U_a} = \frac{U_{H(a)}}{U_{1(a)}},\ THD_{U_b} = \frac{U_{H(b)}}{U_{1(b)}},\ THD_{U_c} = \frac{U_{H(c)}}{U_{1(c)}}$$

$$THD_{I_a} = \frac{I_{H(a)}}{I_{1(a)}},\ THD_{I_b} = \frac{I_{H(b)}}{I_{1(b)}},\ THD_{I_c} = \frac{I_{H(c)}}{I_{1(c)}}$$

where $U_H^2 = \sum_{v=2}^{15} U_v^2$ and $I_H^2 = \sum_{v=2}^{15} I_v^2$ are harmonic components of U (I).

Distortion Factor (per phase)

$$DF_{U_a} = \frac{U_{1(a)}}{U_a},\ DF_{U_b} = \frac{U_{1(b)}}{U_b},\ DF_{U_c} = \frac{U_{1(c)}}{U_c}$$

$$DF_{I_a} = \frac{I_{1(a)}}{I_a},\ DF_{I_b} = \frac{I_{1(b)}}{I_b},\ DF_{I_c} = \frac{I_{1(c)}}{I_c}$$

Equivalent THD

$$THD_{eU} = \frac{U_{eH}}{U_{e1}},\ THD_{eI} = \frac{I_{eH}}{I_{e1}}$$

where $U_{e1}^2 = \frac{U_{a1}^2 + U_{b1}^2 + U_{c1}^2}{3}$, $I_{e1}^2 = \frac{I_{a1}^2 + I_{b1}^2 + I_{c1}^2}{3}$ are the fundamental components of U_e (I_e)

and $U_{eH} = \sqrt{U_e^2 - U_{e1}^2}$, $I_{eH} = \sqrt{I_e^2 - I_{e1}^2}$ are harmonic components of U_e (I_e).

Harmonic Distortion Analysis			
Quantity	Induction Motor	3-phase Rectifier	Single-phase Rectifier
THD(U_{aN})	0	0.040	0.055
THD(U_{bN})	0	0.039	0.055
THD(U_{cN})	0	0.037	0
THD(I_a)	0	0.827	0.954
THD(I_b)	0	0.820	0.954
THD(I_c)	0	0.820	-
DF(U_{aN})	1	0.873	0.835
DF(U_{bN})	1	0.909	0.834
DF(U_{cN})	1	0.893	-
DF(I_a)	1	0.769	0.723
DF(I_b)	1	0.772	0.723
DF(I_c)	1	0.769	-
U_e(1-st)	219.8	215.0	215.4
I_e(1-st)	9.81	8.13	10.78
THD(U_e)	0.018	0.036	0.043
THD(I_e)	0.065	0.833	0.980

Power measurement

Active power

Active power (per phase)

$$P_a = \frac{1}{T}\int_0^T p_a(t)dt = \frac{1}{T}\int_0^T u_a(t)\cdot i_a(t)dt\,,\; P_b = \frac{1}{T}\int_0^T u_b(t)\cdot i_b(t)dt\,,\; P_c = \frac{1}{T}\int_0^T u_c(t)\cdot i_c(t)dt$$

Total Active power

$$P = P_a + P_b + P_c$$

Active power calculation can be done also with utilizing FFT. If you know phase shift between voltage and current of each harmonic and their RMS value then active power per phase is

$$P_{a\Sigma} = P_{a(1)} + \sum_{v=2}^{\infty} P_{a(v)} = U_{a(1)} I_{a(1)} \cos\varphi_{a(1)} + \sum_{v=2}^{\infty} U_{a(v)} I_{a(v)} \cos\varphi_{a(v)}$$

In the simulation only dominant harmonics (1-st, 3-rd, 5-th, 7-th) were used for the calculation.

$$P_{a\Sigma} = P_{a(1)} + P_{a(3)} + P_{a(5)} + P_{a(7)}\,,\; P_{b\Sigma} = P_{b(1)} + P_{b3)} + P_{b(5)} + P_{b(7)}\,,\; P_{c\Sigma} = P_{c(1)} + P_{c(3)} + P_{c(5)} + P_{c(7)}$$

Total active power (using FFT) $P_{\Sigma} = P_{a\Sigma} + P_{b\Sigma} + P_{c\Sigma}$.

Active power on the DC side of rectifier (not present for induction motor)

$$P_{DC} = U_{DC} I_{DC}$$

Active Power Measurement [Watts]			
Power	Induction Motor	3-phase Rectifier	Single-phase Rectifier
P_a	40.5	1709.0	2352.0
P_b	41.6	1709.0	2264.0
P_c	39.9	1709.0	0
P	122.1	5126.0	4617.0
$P_{a\Sigma}$	40.5	1709.0	7059.0
$P_{b\Sigma}$	41.6	1709.0	6796.0
$P_{c\Sigma}$	39.9	1709.0	0
P_{Σ}	122.1	5128.0	4618.0
P_a(1-st)	40.5	1736.0	2451.0
P_b(1-st)	41.6	1736.0	2363.0
P_c(1-st)	39.9	1736.0	0
P(1-st)	122.1	5209.0	4815.0
P_{DC}	none	5103.0	4595.0

Reactive power

Very often used definition for reactive power calculation is

$$Q = \frac{1}{T}\int_0^T u(t-\frac{T}{4})i(t)dt$$

$$Q_{a(T/4)} = \frac{1}{T}\int_0^T u_a(t-\frac{T}{4})i_a(t)dt\ ,\ Q_{b(T/4)} = \frac{1}{T}\int_0^T u_b(t-\frac{T}{4})i_b(t)dt\ ,\ Q_{c(T/4)} = \frac{1}{T}\int_0^T u_c(t-\frac{T}{4})i_c(t)dt$$

$$Q_{(T/4)} = Q_{a(T/4)} + Q_{b(T/4)} + Q_{c(T/4)}$$

Similarlly to active power calculation the FFT can be applied for reactive power calculation

$$Q_{a\Sigma} = Q_{a(1)} + \sum_{v=2}^{\infty} Q_{a(v)} = U_{a(1)} I_{a(1)} \sin\varphi_{a(1)} + \sum_{v=2}^{\infty} U_{a(v)} I_{a(v)} \sin\varphi_{a(v)}$$

$$Q_{a\Sigma} = Q_{a(1)} + Q_{a(3)} + Q_{a(5)} + Q_{a(7)}\ , Q_{b\Sigma} = Q_{b(1)} + Q_{b3)} + Q_{b(5)} + Q_{b(7)}\ ,$$

$$Q_{c\Sigma} = Q_{c(1)} + Q_{c(3)} + Q_{c(5)} + Q_{c(7)}$$

Total reactive power (using FFT) $Q_{\Sigma} = Q_{a\Sigma} + Q_{b\Sigma} + Q_{c\Sigma}$.

Reactive Power Measurement [VAr]			
Power	Induction Motor	3-phase Rectifier	Single-phase Rectifier
Q_a(T/4)	2158.0	190.1	-1375.0
Q_b(T/4)	2159.0	189.7	1527.0
Q_c(T/4)	2160.0	189.1	0
Q(T/4)	6477.0	568.9	152.3
$Q_{a\Sigma}$	2158.0	158.9	-1498.0
$Q_{b\Sigma}$	2159.0	158.8	1402.0
$Q_{c\Sigma}$	2160.0	158.9	0
Q_{Σ}	6478.0	476.5	-96.0

Distortion power

For a calculation of distortion power was used this equation

$$D_a^{\ 2} = \left(U_{1(a)} I_{H(a)}\right)^2 + \left(U_{H(a)} I_{1(a)}\right)^2 + \left(U_{H(a)} I_{H(a)}\right)^2$$

$$D_b^{\ 2} = \left(U_{1(b)} I_{H(b)}\right)^2 + \left(U_{H(b)} I_{1(b)}\right)^2 + \left(U_{H(b)} I_{H(b)}\right)^2$$

$$D_c^{\ 2} = \left(U_{1(c)} I_{H(c)}\right)^2 + \left(U_{H(c)} I_{1(c)}\right)^2 + \left(U_{H(c)} I_{H(c)}\right)^2$$

$$D = D_a + D_b + D_c$$

Distortion Power Measurement [VA]			
Power	Induction Motor	3-phase Rectifier	Single-phase Rectifier
D_a	6.1	1458.0	2777.0
D_b	8.8	1458.0	2753.0
D_c	5.7	1458.0	0
D	20.7	4373.0	5530.0

Apparent power

There is a number of definitions for an apparent power calculation. Some of them were used in the simulation.
One of these definitions employ Pythagoras relation between active and reactive power. There is no problem to measure active power P accuratelly, consequently accuracy of calculation of S depends on accuracy of Q. This definition is often used by power-transmission companies

$$S^2_{a(Pyth)} = P^2_{a(Pyth)} + Q^2_{a(Pyth)},\ S^2_{b(Pyth)} = P^2_{b(Pyth)} + Q^2_{b(Pyth)},\ S^2_{c(Pyth)} = P^2_{c(Pyth)} + Q^2_{c(Pyth)}$$

$$S_{(Pyth)} = S_{a(Pyth)} + S_{b(Pyth)} + S_{c(Pyth)}$$

Another definition is called "arithmetic" apparent power S_A

$$S_A = S_{a(UI)} + S_{b(UI)} + S_{c(UI)}$$

where $S_{a(UI)} = U_a I_a$, $S_{b(UI)} = U_b I_b$, $S_{c(UI)} = U_c I_c$.

FFT technique can be used in the same way as for P and Q computation

$$S_{a\Sigma} = S_{a(1)} + \sum_{v=2}^{\infty} S_{a(v)} = U_{a(1)} I_{a(1)} + \sum_{v=2}^{\infty} U_{a(v)} I_{a(v)}$$

$$S_{a\Sigma} = S_{a(1)} + S_{a(3)} + S_{a(5)} + S_{a(7)}, S_{b\Sigma} = S_{b(1)} + S_{b3)} + S_{b(5)} + S_{b(7)},$$

$$S_{c\Sigma} = S_{c(1)} + S_{c(3)} + S_{c(5)} + S_{c(7)}$$

Total apparent power (using FFT) $S_\Sigma = S_{a\Sigma} + S_{b\Sigma} + S_{c\Sigma}$.

Vector (or phasor) apparent power

$$S_V{}^2 = S_1^2 + D^2 = P_1^2 + Q_1^2 + D^2$$

Equivalent apparent power

$$S_e = 3U_e I_e$$

Apparent Power Measurement [VA]			
Power	Induction Motor	3-phase Rectifier	Single-phase Rectifier
S_a(Pyth)	2158.4	1719.5	2724.4
S_b(Pyth)	2159.4	1719.5	2730.8
S_c(Pyth)	2160.4	1719.4	0
S(Pyth)	6478.0	5158.0	5455.0
S_a(UI)	2159.0	2279.0	3962.0
S_b(UI)	2160.0	2279.0	3927.0
S_c(UI)	2160.0	2279.0	0
S_A(UI)	6479.0	6837.0	7889.0
$S_{a\Sigma}$	2159.0	1796.0	2964.0
$S_{b\Sigma}$	2160.0	1796.0	2939.0
$S_{c\Sigma}$	2160.0	1796.0	0
S_Σ	6479.0	5387.0	5906.0
S_V	6479.0	6827.0	7335.0
S_e	6479.0	6827.0	9761.0

Power Factor measurement

Common definition for power factor is $PF = \frac{P}{S}$. Therefore, accuracy of *PF* depends on accuracy of *P* and *S*.

$$PF_{a(Pyth)} = \frac{P_a}{S_{a(Pyth)}},\ PF_{b(Pyth)} = \frac{P_b}{S_{b(Pyth)}},\ PF_{c(Pyth)} = \frac{P_c}{S_{c(Pyth)}}$$

$$PF_{(Pyth)} = \frac{P}{S_{(Pyth)}}$$

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$$PF_{a(UI)} = \frac{P_a}{S_{a(UI)}},\ PF_{b(UI)} = \frac{P_b}{S_{b(UI)}},\ PF_{c(UI)} = \frac{P_c}{S_{c(UI)}}$$

$$PF_{(UI)} = \frac{P_a}{S_{(UI)}}$$

$$PF_V = \frac{P_a}{S_V}$$

Displacement power factor is defined as

$DPF = \frac{P_1}{S_1} = \cos\varphi_1$, where φ_1 is a phase shift between fundamental components of U and I.

Power Factor Measurement			
Quantity	Induction Motor	3-phase Rectifier	Single-phase Rectifier
PF_a(Pyth)	0.019	0.994	0.863
PF_b(Pyth)	0.019	0.994	0.829
PF_c(Pyth)	0.019	0.994	-
PF(Pyth)	0.019	0.994	0.846
PF_a(UI)	0.019	0.749	0.593
PF_b(UI)	0.019	0.749	0.576
PF_c(UI)	0.019	0.749	-
PF_A(UI)	0.019	0.749	0.585
PF_V	0.019	0.751	0.629
PF_e	0.019	0.751	0.473
DPF	0.019	0.994	0.868

Summary & Conclusions

The main reason of writing this document was to analyze the contemporary trends of electric energy measurement and monitoring. The large part includes the summary of the theory which is necessary for the majority of contemporary methods of the measurement of electrical power and energy. The experts dealing with this topic differ in the opinion of the right method of measurement and the analysis of the consumption of the electric energy. Old and current methods result from the anticipation that currents and voltages are sine-shaped. In the latest time, there is installed more and more devices in the network causing a large volume of harmonics in the lines. That is why the values measured by traditional instruments do not represent the real energy consumption.
The methods of and approaches to the measurement of reactive and apparent power is in the last years a very discussed topic in the expert papers.

The comparison of results of the different approaches has been carried out by simulation. The three methods were applied in the program MATLAB/Simulink for three different types of load. Induction motor represented linear, symmetrical load; Three-phase rectifier represented symmetrical, non-linear load, and Single-phase rectifier represented non-linear, non-symmetrical load.
The results shown in tables (Chapter Simulation & Modeling) clearly confirm that the main divergences in the measurement of the same quantities by the different methods come in non-linear, non-symmetrical, unbalanced loads.
Active power is clearly defined and its measurement is accurate and simple. But the problem is much more difficult in terms of defining reactive and apparent energy and power factor as well.
The results of simulation confirm that the different methods bring different results. The difference is the most clearly seen for non-linear and non-symmetrical consumption (Single-phase rectifier) and reaches almost 40% (S_e=9761VA; $S_{(Pyth)}$=5455VA).

Manufacturers of reactive power measurement devices use different methods for evaluation and that is why they reach different results. It seems to take some time while the independent scientists, measuring instrument's manufacturers and engineers of electrical companies agree on particular definitions and the equations of quantities describing electrical energy consumption.

Metering has always been an artful compromise between technical capabilities, cost of meters, and theoretical considerations. The compromises selected for prior-generation revenue meters were accurate for sinusoidal voltages and currents. But loads today are increasingly non-linear, and digital sampling meter technology holds out the promise of measuring a range of quantities that were unavailable in the past[13].

Suggestions and recommendations for further research

The results of simulation of different methods of P&E measurement differ very much (mainly in the case of apparent and reactive power). That is why it would be useful to realize a microprocessor-based measuring instrument (in the labs of CTU Prague) and compare our results with the measurement obtained by professional instruments (collaboration with Dept.of measurement, Dept.of Energy, Dept.of Technology – CTU Prague).
It would be very useful and reasonable to focus on the measurement methods using the FFT technique, because there are already done algorithms for FFT used for IQPreAlert.

The economic issues of power loses and power factor in polyphase systems will become increasingly important in the future. There is need to address these issues in a practical manner that can be implemented with existing technology.

Glossary of Power Quality Terms

The purpose of this section is to present concise definitions of many common power quality terms. For the most part, these definitions coincide with current industry efforts to define power quality terms.

Active Filter. Any of a number of sophisticated power electronic devices for eliminating harmonic distortion.

Common Mode Voltage. The noise voltage that appears equally from current-carrying conductor to ground.

Coupling. Circuit element or elements, or network, that may be considered common to the input mesh and the output mesh and through which energy may be transferred from one to another.

Crest Factor. A value reported by many power quality monitoring instruments representing the ratio of the crest value of the measured waveform to the rms of the fundamental. For example, the crest factor of a sinusoidal wave is 1.414.

Critical Load. Devices and equipment whose failure to operate satisfactorily jeopardizes the health or safety of personnel, and/or results in loss of function, financial loss, or damage to property deemed critical by the user.

Current Distortion. Distortion in the ac line current. See Distortion.

Differential Mode Voltage. The voltage between any two of a specified set of active conductors.

Dip. See Sag.

Distortion. Any deviation from the normal sine wave for an ac quantity.

Dropout. A loss of equipment operation (discrete data signals) due to noise, sag, or interruption.

Dropout Voltage. The voltage at which a device will release to its de-energized position (for this document, the voltage at which a device fails to operate).

Electromagnetic Compatibility. The ability of a device, equipment or system to function satisfactorily in its electromagnetic environment without introducing intolerable electromagnetic disturbances to anything in that environment.

Failure Mode. The effect by which failure is observed.

Fault. Generally refers to a short circuit on the power system.

Fault, Transient. A short circuit on the power system usually induced by lightning, tree branches, or animals which can be cleared by momentarily interrupting the current.

Flicker. Impression of unsteadiness of visual sensation induced by a light stimulus whose luminance or spectral distribution fluctuates with time.

Frequency Deviation. An increase or decrease in the power frequency. The duration of a frequency deviation can be from several cycles to several hours.

Frequency Response. In power quality usage, generally refers to the variation of impedance of the system, or a metering transducer, as a function of frequency.

Fundamental (Component). The component of order 1 (50 to 60 Hz) of the Fourier series of a periodic quantity.

Ground. A conducting connection, whether intentional or accidental, by which an electric circuit or equipment is connected to the earth, or to some conducting body of relatively large extent that serves in place of the earth. Note: It is used for establishing and maintaining the potential of the earth (or of the conducting body) or approximately that potential, on conductors connected to it, and for conducting ground currents to and from earth (or the conducting body).

Ground Electrode. A conductor or group of conductors in intimate contact with the earth for the purpose of providing a connection with the ground.

Ground Grid. A system of interconnected bare conductors arranged in a pattern over a specified area and on or buried below the surface of the earth. The primary purpose of the ground grid is to provide safety for workmen by limiting potential differences within its perimeter to safe levels in case of high currents which could flow if the circuit being worked became energized for any reason or if an adjacent energized circuit faulted. Metallic surface mats and gratings are sometimes utilized for the same purpose . This is not necessarily the same as a Signal Reference Grid.

Ground Loop. A potentially detrimental loop formed when two or more points in an electrical system that are nominally at ground potential are connected by a conducting path such that either or both points are not at the same ground potential.

Ground Window. The area, through which, all grounding conductors, including metallic raceways enter a specific area. It is often used in communications systems through which the building grounding system is connected to an area that would otherwise have no grounding connection.

Harmonic (component). A component of order greater than one of the Fourier series of a periodic quantity.

Harmonic Content. The quantity obtained by subtracting the fundamental component from an alternating quantity.

Harmonic Distortion. Periodic distortion of the sine wave. See Distortion and Total Harmonic Distortion (THD).

Harmonic Filter. On power systems, a device for filtering one or more harmonics from the power system. Most are passive combinations of inductance, capacitance, and resistance. Newer technologies include active filters that can also address reactive power needs.

Harmonic Number. The integral number given by the ratio of the frequency of a harmonic to the fundamental frequency.

Harmonic Resonance. A condition in which the power system is resonating near one of the major harmonics being produced by nonlinear elements in the system, thus exacerbating the harmonic distortion.

Interharmonic (component). A frequency component of a periodic quantity that is not an integer multiple of the frequency at which the supply system is designed to operate (e.g. 50 Hz or 60 Hz).

Isolation. Separation of one section of a system from undesired influences of other sections.

Linear Load. An electrical load device which, in steady state operation, presents an essentially constant load impedance to the power source throughout the cycle of applied voltage.

Long Duration Variation. A variation of the rms value of the voltage from nominal voltage for a time greater than one minute. Usually further described using a modifier indicating the magnitude of a voltage variation (e.g., Undervoltage, Overvoltage, or Voltage Interruption).

Noise. Unwanted electrical signals which produce undesirable effects in the circuits of the control systems in which they occur. (For this document, "control systems" is intended to include sensitive electronic equipment in total or in part.)

Nominal Voltage. (Vn). A nominal value assigned to a circuit or system for the purpose of conveniently designating its voltage class (as 208/120, 480/277, 600).

Nonlinear Load. Electrical load which draws current discontinuously or whose impedance varies throughout the cycle of the input ac voltage waveform.

Normal Mode Voltage. A voltage that appears between or among active circuit conductors.

Notch. A switching (or other) disturbance of the normal power voltage waveform, lasting less than a half-cycle; which is initially of opposite polarity than the waveform, and is thus subtracted from the normal waveform in terms of the peak value of the disturbance voltage. This includes complete loss of voltage for up to a half cycle.

Overvoltage. When used to describe a specific type of long duration variation, refers to a voltage having a value of at least 10% above the nominal voltage for a period of time greater than 1 minute.

Passive Filter. A combination of inductors, capacitors, and resistors designed to eliminate one or more harmonics. The most common variety is simply an inductor in series with a shunt capacitor, which short-circuits the major distorting harmonic component from the system.

Phase Shift. The displacement in time of one voltage-waveform relative to other voltage-waveform(s).

Power Factor, Displacement. The power factor of the fundamental frequency components of the voltage and current wave forms.

Power Factor (True). The ratio of active power (watts) to apparent power (voltamperes).

Pulse. An abrupt variation of short duration of a physical quantity followed by a rapid return to the initial value.

Reclosing. The common utility practice on overhead lines of closing the breaker within a short time after clearing a fault taking advantage of the fact that most faults are transient, or temporary.

Recovery Time. Time interval needed for the output voltage or current to return to a value within the regulation specification after a step load or line change. Also may indicate the time interval required to bring a system back to its operating condition after an interruption or dropout.

Recovery Voltage. The voltage that occurs across the terminals of a pole of a circuit interrupting device upon interruption of the current.

Safety Ground. See: Equipment Grounding Conductor.

Sag. A decrease to between 0.1 and 0.9 pu in rms voltage or current at the power frequency for durations of 0.5 cycles to one minute.

Shield. As normally applied to instrumentation cables, refers to a conductive sheath (usually metallic) applied, over the insulation of a conductor or conductors, for the purpose of providing means to reduce coupling between the conductors so shielded and other conductors which may be susceptible to, or which may be generating unwanted electrostatic or electromagnetic fields (noise).

Shielding. Shielding is the use of a conducting and/or ferromagnetic barrier between a potentially disturbing noise source and sensitive circuitry. Shields are used to protect cables (data and power) and electronic circuits. They may be in the form of metal barriers, enclosures, or wrappings around source circuits and receiving circuits.

Shielding (of utility lines). The construction of a grounded conductor or tower over the lines to intercept lightning strokes in an attempt to keep the lightning currents out of the power system.

Swell. A temporary increase in the rms value of the voltage of more than 10% the nominal voltage, at the power frequency, for durations from 0.5 cycle to one minute.

Total Demand Distortion (TDD). The ratio of the root-mean- square of the harmonic current to the root-mean-square value of the rated or maximum demand fundamental current, expressed as a percent.

Total Disturbance Level The level of a given electromagnetic . disturbance caused by the superposition of the emission of all pieces of equipment in a given system.

Total Harmonic Distortion (THD). The ratio of the root-mean- square of the harmonic content to the root-mean-square value of the fundamental quantity, expressed as a percent of the fundamental.

Transient. Pertaining to or designating a phenomenon or a quantity which varies between two consecutive steady states during a time interval that is short compared to the time scale of interest. A transient can be a unidirectional impulse of either polarity or a damped oscillatory wave with the first peak occurring in either polarity.

Triplen Harmonics. A term frequently used to refer to the odd multiples of the third harmonic, which deserve special attention because of their natural tendency to be zero sequence.

Undervoltage. When used to describe a specific type of long duration variation, refers to a measured voltage having a value at least 10% below the nominal voltage for a period of time greater than one minute.

Voltage Change. A variation of the rms or peak value of a voltage between two consecutive levels sustained for definite but unspecified durations.

Voltage Dip. See Sag.

Voltage Distortion. Distortion of the ac line voltage. See Distortion.

Voltage Fluctuation. A series of voltage changes or a cyclical variation of the voltage envelope.

Voltage Imbalance (Unbalance). A condition in which the three phase voltages differ in amplitude or are displaced from their normal 120 degree phase relationship or both. Frequently expressed as the ratio of the negative sequence or zero sequence voltage to the positive sequence voltage, in percent.

Voltage Interruption. Disappearance of the supply voltage on one or more phases. Usually qualified by an additional term indicating the duration of the interruption (e.g., Momentary, Temporary, or Sustained.)

Voltage Regulation. The degree of control or stability of the rms voltage at the load. Often specified in relation to other parameters, such as input-voltage changes, load changes, or temperature changes.

Voltage Magnification. The magnification of capacitor switching oscillatory transient voltage on the primary side by capacitors on the secondary side of a transformer.

Waveform Distortion. A steady state deviation from an ideal sine wave of power frequency principally characterized by the spectral content of the deviation.

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Web pages

[W1]	www.doe.gov	-	*The U.S. Department of Energy Home Page*
[W2]	www.navy.mil	-	*The official web site of the United States Navy.*
[W3]	www.ase.org	-	*The Alliance to Save Energy Home Page*
[W4]	www.millennianet.com/valhalla	-	*Valhalla Scientific's WWW server*
[W5]	www.ab.com	-	*Allen-Bradley Home Page*
[W6]	www.summittechnology.com	-	*Summit Technology - makers of PowerSight, the tool of first resort for Energy Analysis*
[W7]	www.pqnet.electrotek.com	-	*The PQ Network - service to the power quality community*
[W8]	www.powermeasurement.com	-	*Power Measurement - The manufacturer of advanced, multi-function digital power meters and billing meters*

Appendix:

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